$$b_m(x) = \frac{2}{w}\int_0^w f(x,y)\sin\left(\frac{m\pi y}{w}\right)dy$$

$$A_{nm} = \frac{2}{L}\int_0^L b_m(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$A_{nm} = \frac{2}{L}\int_0^L \left(\frac{2}{w}\int_0^w f(x,y)\sin\left(\frac{m\pi y}{w}\right)dy\right)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$\boxed{A_{nm} = \frac{4}{Lw}\int_0^L\int_0^w f(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{w}\right)dy\,dx}$$

$$u_t(x,y,0) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\sqrt{\lambda_{nm}}\,c\,B_{nm}\right)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{w}\right) = g(x,y)$$

$$\left(\sqrt{\lambda_{nm}}\,c\,B_{nm}\right) = \frac{4}{Lw}\int_0^L\int_0^w g(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{w}\right)dy\,dx$$

$$\boxed{B_{nm} = \frac{4}{Lwc\sqrt{\lambda_{nm}}}\int_0^L\int_0^w g(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{w}\right)}$$