$$u(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} u_{nm}(x,y,t)$$

General Solution...

$$u(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left(A_{nm} \cos\sqrt{\lambda_{nm}}\, ct + B_{nm} \sin\sqrt{\lambda_{nm}}\, ct \right) \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi y}{w}\right)$$

$$\sqrt{\lambda_{nm}} = \sqrt{\left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{w}\right)^2} = \pi\sqrt{\left(\frac{n}{L}\right)^2 + \left(\frac{m}{w}\right)^2} = \nu_{nm}$$

Initial Conditions: $u(x,y,0) = f(x,y)$

$u_t(x,y,0) = g(x,y)$

$$u(x,y,0) = f(x,y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \sin\frac{n\pi x}{L} \sin\frac{m\pi y}{w}$$

$$= \sum_{m=1}^{\infty} \underbrace{\left[\sum_{n=1}^{\infty} A_{nm} \sin\frac{n\pi x}{L} \right]}_{b_m(x)} \sin\frac{m\pi y}{w}$$

$$b_m(x) = \sum_{n=1}^{\infty} A_{nm} \sin\frac{n\pi x}{L}$$

$$f(x,y) = \sum_{m=1}^{\infty} b_m(x) \sin\frac{m\pi y}{w}$$