

2-D Wave Equation Formulas

$$\begin{array}{c}
 u=0 \\
 \omega \\
 \begin{array}{|c|}
 \hline
 u_{tt} = c^2 \nabla^2 u \\
 \hline
 \end{array} \\
 u=0 \\
 0 \qquad u=0 \qquad L
 \end{array}$$

$$T'' + \lambda c^2 T = 0$$

$$X'' + \mu X = 0$$

$$Y'' + (\lambda - \mu) Y = 0$$

$$X(0) = 0$$

$$Y(0) = 0$$

$$X(L) = 0$$

$$Y(\omega) = 0$$

$$X_n(x) = \sin \frac{n\pi x}{L}$$

$$\mu_n = \left(\frac{n\pi}{L}\right)^2, \quad n = 1, 2, 3, \dots$$

$$Y(y) = \sin \frac{m\pi y}{\omega}$$

$$(\lambda - \mu)_m = \left(\frac{m\pi}{\omega}\right)^2, \quad m = 1, 2, 3, \dots$$

$$\begin{array}{c}
 (\lambda - \mu_n)_m = \left(\frac{m\pi}{\omega}\right)^2 \Rightarrow \lambda_{n,m} = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{\omega}\right)^2 > 0 \\
 \downarrow \\
 \left(\frac{n\pi}{L}\right)^2
 \end{array}$$

If $\lambda > 0$ we get oscillation in time (cosines & sines)
 If $\lambda < 0$ we get exponential growth/decay.

$$T_{nm}(t) = A_{nm} \cos \sqrt{\lambda_{nm}} ct + B_{nm} \sin \sqrt{\lambda_{nm}} ct$$

$$U_{nm}(x, y, t) = T_{nm}(t) X_n(x) Y_m(y)$$