

Try $u(x, y, t) = X(x) Y(y) T(t)$

$$\frac{\partial^2}{\partial t^2} (XYT) = c^2 \left[\frac{\partial^2}{\partial x^2} (XYT) + \frac{\partial^2}{\partial y^2} (XYT) \right] \bigg/ c^2 XYT$$

$$\frac{T''}{c^2 T} = \frac{X''}{X} + \frac{Y''}{Y} = -\lambda \text{ (say)}$$

independent $x, y =$ independent $t =$ constant

$$\frac{X''}{X} = -\frac{Y''}{Y} - \lambda = -\mu$$

independent $y, t =$ independent $x, t =$ constant

$$T'' + \lambda c^2 T = 0 \quad \left| \quad \begin{array}{l} X'' + \mu X = 0 \\ X(0) = 0 \\ X(L) = 0 \end{array} \quad \left| \quad \begin{array}{l} Y'' + (\lambda - \mu) Y = 0 \\ Y(0) = 0 \\ Y(w) = 0 \end{array} \right.$$

B.C.'s: $u(0, y, t) = 0 = X(0) Y(y) T(t)$ $\left. \begin{array}{l} Y(y) = 0 \\ T(t) = 0 \end{array} \right\}$ Trivial
 $\boxed{X(0) = 0}$