

1. Begin with the Taylor series  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$  and mimic Euler's work to derive the sum of the reciprocals of the squares of the odd integers:

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81} + \dots = \frac{\pi^2}{8}.$$

Now use the Taylor expansion for  $\cos x$  above and a clever choice for  $x$  to show that

$$\sqrt{2} = \frac{2 \cdot 2 \cdot 6 \cdot 6 \cdot 10 \cdot 10 \cdot 14 \cdot 14 \cdot \dots}{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15 \cdot \dots}.$$