Show that if $X \in Cauchy(0,1)$, then so is $\frac{1}{X}$

So far I have:

Set $Z = \frac{1}{X}$. Note: $f(x) = \frac{1}{\pi} \quad \frac{1}{1+x^2}, \qquad -\infty < x < \infty$

$F(z) = P[Z \leq z] = P[\frac{1}{X} \leq z] = P[X \leq z^3] = \int\limits_{?}^{z^3} \frac{1}{\pi} \frac{1}{1+x^2} dx$

$f(z) = \frac{d}{dz} \left(\int\limits_{?}^{z^3} \frac{1}{\pi} \frac{1}{1+z^2} dx \right)$

That's where I'm stuck!!!