1.) Given the wave equation below (with *variable* tension τ(x) = x) and appropriate Boundary Conditions:

 , 0 ≤ x ≤ 1 , 0 ≤ t

u(0,t) & ux(0,t) finite , u(1,t) = 0

(a) Apply Separation of Variables to the PDE to get 2 ODE’s.

(b) Show that the Eigenvalue problem is a *singular* Sturm-Liouville system; identify the functions p(x), w(x), and q(x). **Do not try to solve it.**

(c) Based on the equation below, called the “Rayleigh quotient”, do you expect this problem to have any *negative* Eigenvalues? How about a *zero* Eigenvalue? Be sure to use specific p(x), w(x), q(x) and Boundary Conditions for this problem.

