Consider the function of two real variables x and y ($y \neq 1$) defined by

$$f(x,y) = x^2 - y^2 - \frac{4x^2}{(y-1)^2}$$

(i) Find the first-order and second-order partial derivatives of f.

(ii) Determine the second-order Taylor polynomial for f near $(0,-1)$. (There is no need to expand any brackets that may appear in your answer.)

(iii) The value of f when $x = -1$ and $y = 0$ is -3. Suppose that the maximum absolute error in the value of x is 0.01, whereas the maximum absolute error in the value of y is 0.03. Use first-order partial derivatives to determine the least and greatest possible values that $f(-1,0)$ may take, and hence the maximum possible error in taking the value $f(-1,0) = -3$.

(iv) Use the chain rule to determine, in terms of t, the rate of change of f with time along the path given at time t by $x(t) = \tan t$, $y(t) = 1 + \sec t$, simplifying your answer as far as possible. (Hint: when simplifying, express each term in sines and cosines.)

(v) Locate and classify all the stationary points of f.