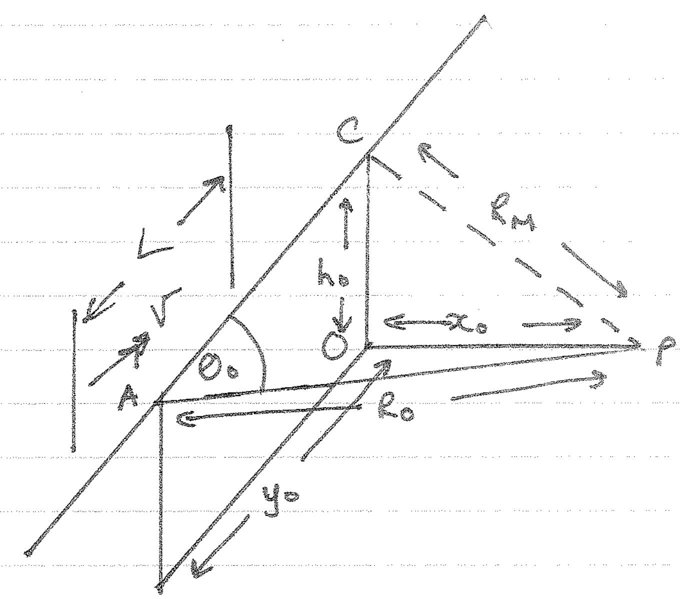
SYNTHETIC APERTURE RADAR

Synthetic aperture is a radar technique, implemented from aircraft or spacecraft, in which an "effective antenna aperture" is synthesised that is much longer than the real antenna. Radar motion is a prerequisite of the SAR technique.

Phase & Frequency history of a point return

The phase history of a ground return can be deduced from the range history to a specific point on the ground during the aircraft flight through the observation section.

Consider an aircraft flying at height h_0 along the y axis (assume flat Earth). The point of interest is P, and we choose the origin O to be beneath the point of closest approach.



The distance P from the subtrack is x_0. Time $t=0$ corresponds to the center of the observation period. The range from the radar to P at time $t=0$ is

$$R_0 = \left(x_0^2 + y_0^2 + h_0^2\right)^{1/2}$$

The angle between the aircraft velocity vector V and the radar range vector to P is θ_0, obtained from

$$\cos\theta_0 = \frac{y_0}{R_0}.$$

The angle θ_0 is called the squint angle.

The observation section marked L is the section of the aircraft path during which the return from point P is processed. Clearly, all along L the point P needs to be within the beam of the real antenna. The observation time is given by

$$T = \frac{L}{V} \qquad V = \text{constant aircraft vel}^{y}.$$

During T, the range equation is

$$R = \left[x_0^2 + (y_0 - Vt)^2 + h_0^2\right]^{1/2} \qquad -T/2 \leq t \leq T/2.$$

$$\equiv \quad R = \left[R_0^2 - 2R_0Vt\cos\theta_0 + V^2t^2\right]^{1/2}$$

$$\text{since} \quad R_0^2 = x_0^2 + y_0^2 + h_0^2$$

Performing a Taylor series expansion on

$$R = R_0\left[1 + \left(\frac{V^2t^2 - 2R_0Vt\cos\theta_0}{R_0^2}\right)\right]^{1/2}$$

we obtain

$$R = R_0\left\{1 + \frac{1}{2}\left(\frac{V^2t^2 - 2R_0Vt\cos\theta_0}{R_0^2}\right) + \frac{1}{2!}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{V^2t^2 - 2R_0Vt\cos\theta}{R_0^2}\right)^2 + \dots\right\}$$

$$= R_0 \left\{ 1 + \frac{V^2 t^2 - 2R_0 V t \cos\theta}{2R_0^2} - \frac{1}{8}\left(\frac{V^2 t^4 - 4R_0 V^3 t^2 \cos\theta + 4R_0^2 V^2 t^2 \cos^2\theta}{R_0^4} \right) + \cdots \right\}$$

$$\therefore R = R_0 - Vt\cos\theta + \frac{V^2 t^2}{2R_0}\sin^2\theta + \frac{V^3 t^3}{2R_0^2}\cos\theta_0 \sin^2\theta_0 + \cdots$$

For the special case of $\theta_0 = 90°$ NOTE: we are now at C @ $t=0$.

$$R = R_0 + \frac{V^2 t^2}{2R_0} - \frac{V^4 t^4}{8R_0^3} + \cdots, \quad \theta_0 = 90°$$

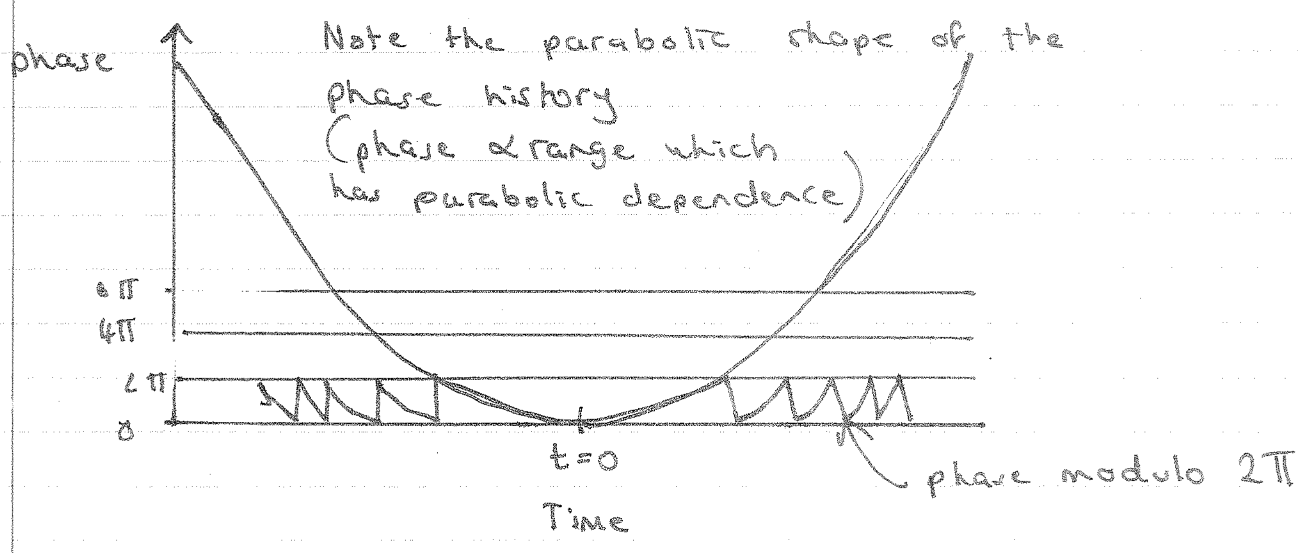
Given that the two-way phase shift is related to range as

$$\phi = 2R \frac{2\pi}{\lambda}$$

at 90° we have

$$\phi = \phi_0 + \frac{2\pi V^2 t^2}{\lambda R_0} - \frac{2\pi V^4 t^4}{4\lambda R_0^3} + \cdots, \quad \theta_0 = 90°$$

A plot for arbitrary θ_0 is given below:



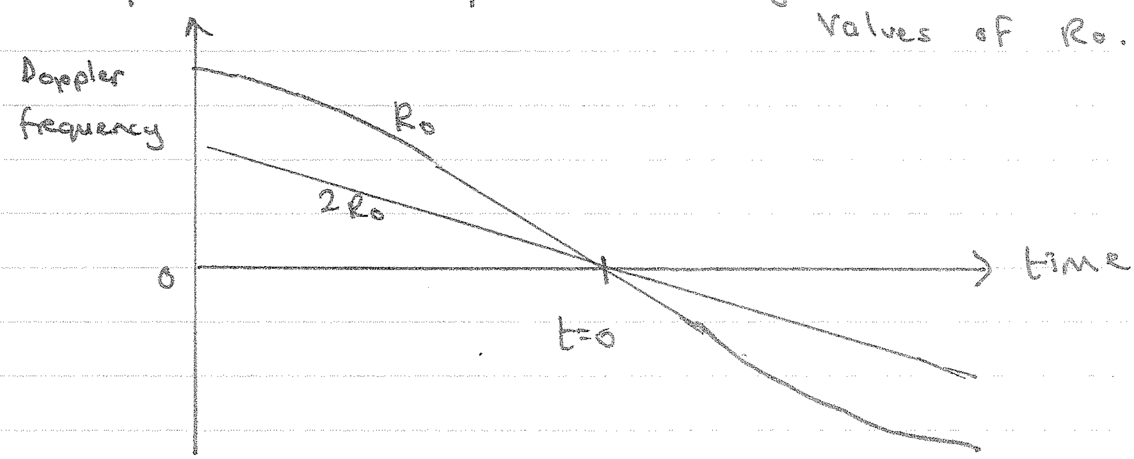
Given that Doppler frequency is related to range by

$$f_D = -\frac{2\dot{R}}{\lambda} \quad \text{where} \quad \dot{R} = \frac{dR}{dt}$$

we get for a squint angle $\theta_0 = 90°$

$$f_D = -\frac{2V^2 t}{\lambda R_0} + \frac{V^4 t^3}{\lambda R_0^3} + \cdots$$

Two plots of this expression are given below for different values of R_0.



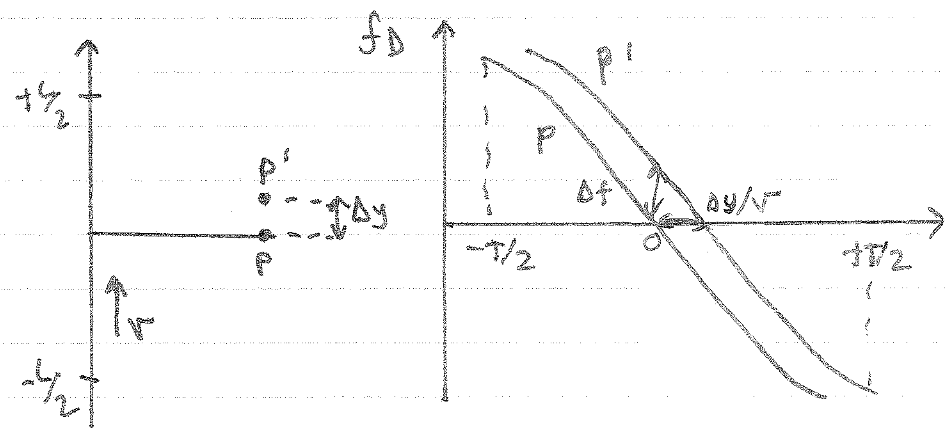
Note that for $\theta_0 = 90°$, at $t=0$ the vector to P is perpendicular to the velocity vector, hence the Doppler is zero.

For antennas not orientated at squint angles of $90°$, the Doppler will obviously differ from non-zero.

Along-track resolution

In this section we will obtain the along-track (y-direction) resolution.

Consider viewing the SAR geometry from above the aircraft track & consider two points on the ground, P & P'.
The two points are separated by Δy. The question is: how small a Δy can SAR resolve?



For SAR with a 90° squint angle, the Doppler history of returns from P & P' are two parallel, descending, nearly straight lines. The frequency separation between the two is ~ constant.

Near the point of closest approach, the frequency difference can be obtained from

$$f_D = -\frac{2V^2 t}{\lambda R_0} + \dots \quad , \ \theta_0 = 90°$$

& given $t = \Delta y / V$, thus

$$\Delta f = \frac{2V}{\lambda R_0} \Delta y$$

The ability of a filter to separate two frequencies Δf apart is related to the signal duration time, which in our case is T, by

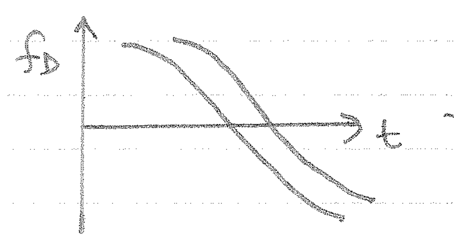
$$\Delta f = \frac{1}{T} = \frac{V}{L}$$

$\therefore$ We can obtain the along-track resolution

$$\Delta y = \frac{\lambda R_0}{2L}.$$

NOTE: For a squint angle different from $\theta_0 = 90°$, we will obtain the same result. Given that R_0 is smallest at $\theta_0 = 90°$, the resolution is minimised for that case.

We emphasize that the processor has to separate (filter) two separate signals whose frequencies are Δf apart - The issue is further complicated given the frequencies are changing ($\sim$ linear descent, ie/



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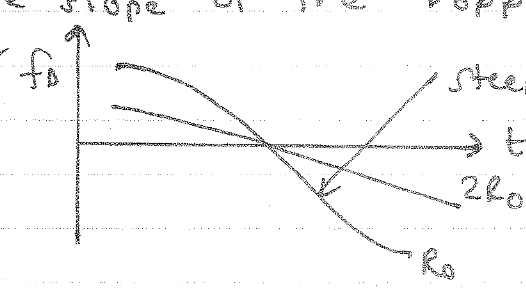
One is therefore required to introduce quadratic phase compensation.

Unfortuneately, different filters are required for different ranges. If point P is farther away from the aircraft sub-track, the slope of the Doppler return will be smaller, ie/



Steeper Doppler at closer range.

In broadside SAR, R_o is the slant range at the point of closest approach C. Because different filters are req^d for different ranges, one could argue the the use of Doppler filtering to resolve range. However, a much better resolution is obtained conventionally in the delay domain.

Although we have so-far considered a continuous phase & frequency from a return signal, the signal is usually pulsed. Consequently, the phase & frequency will be samples from the continuous curves. A pulsed signal yields simple range processing by means of range gates along the delay axis.

SAR — CONTINUED ....

Resolution Limit

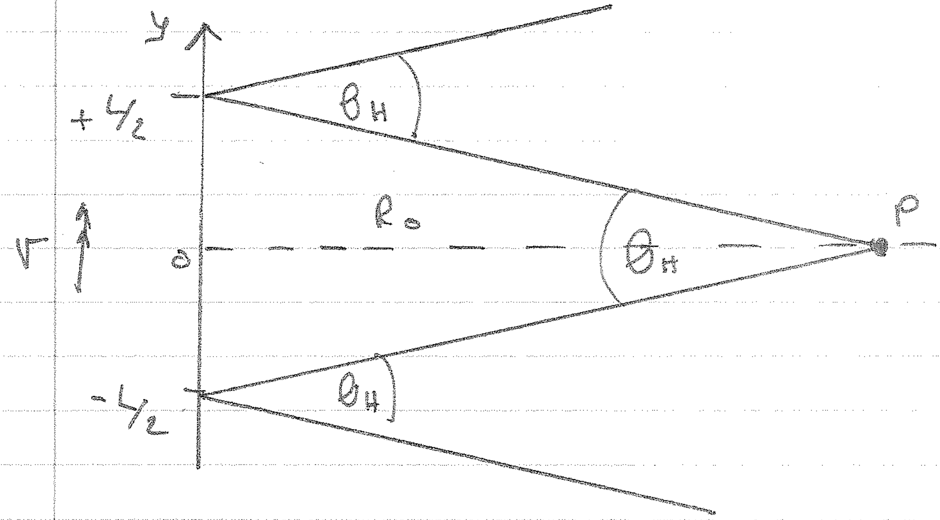
We obtained the along-track resolution as

$$D_y = \frac{\lambda R_0}{2L}.$$

Since D_y is inversely related to the length of the observation section L, increasing L will improve resolution.

This results in the obvious requirement that the point P will be within the antenna beam illumination all through section L.

Consider a horizontal antenna beamwidth θ_H, and let L be extended until P is at the upper edge when the antenna is at $y = -L/2$ and at the lower edge when $y = L/2$.



If $R_0 >> L$, the beamwidth is approximately

$$\theta_H = \frac{L}{R_0}$$

Also, from insert 1B, page 6 in the Levanon text, we have the relation between wavelength, beamwidth & aperture:

$$\theta_H = \frac{\lambda}{d}$$

∴ We obtain $L = \frac{\lambda R_0}{d}$

which given $\Delta y = \frac{\lambda R_0}{2L}$

yields a resolution limit

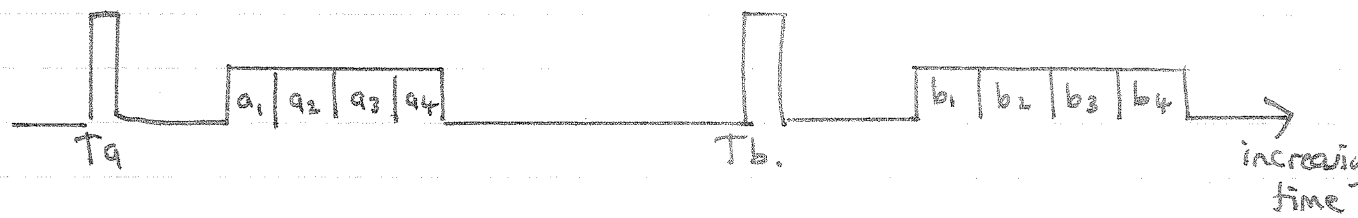
$$\Delta y_{MIN} = \frac{d}{2}.$$

ie, the resolution in the along-track direction is one-half the aperture, ie smaller apertures → increased resolution!

IMPORTANT keep in mind that aperture reduction to the antenna to improve resolution, however, reduces the antenna gain G, and that the return signal power is related to G^2. ie, we have a power-resolution conflict.

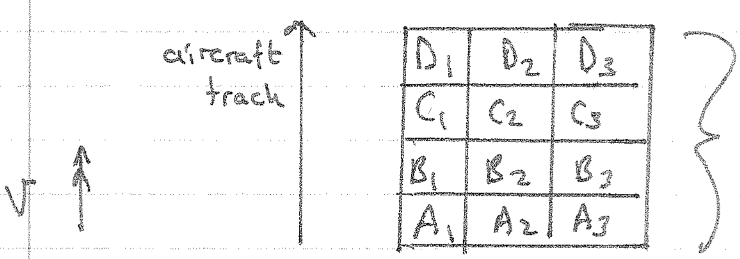
SAR Signal Processing

Consider transmitted pulses $T_a, T_b, \ldots$, each of which is followed by returns $a_1, a_2 \ldots$, $b_1, b_2, \ldots$, giving range information from each pulse. With increasing time this may look like:



The received bins e.g a_1, a_2, a_3, correspond to different land ranges (ie time delays).

If we consider a view from above the aircraft track, these bins may look like

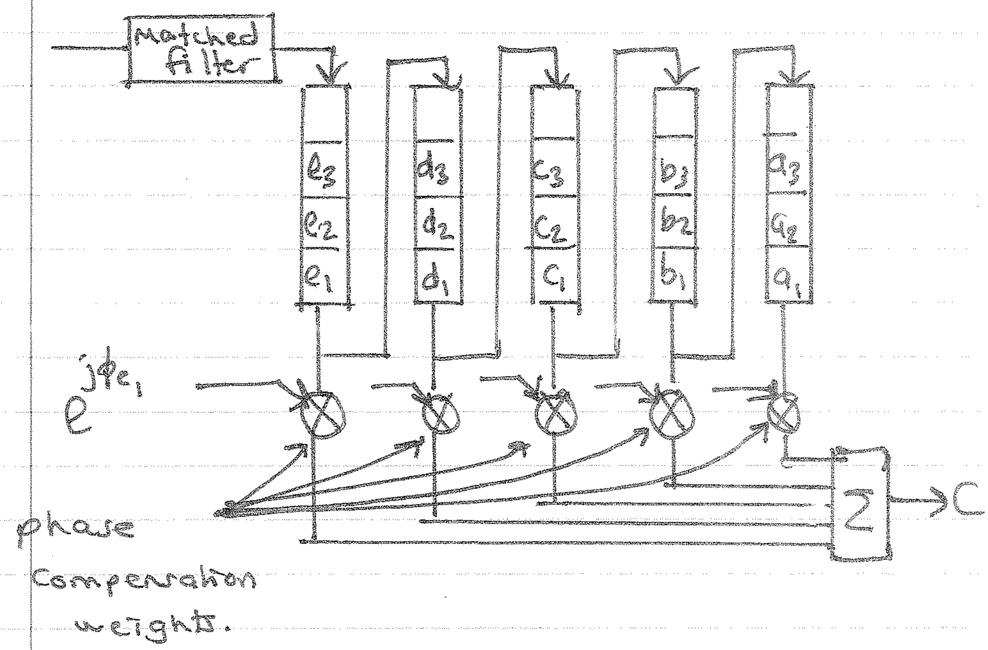


land area corresponding to return pulses a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3 & d_1, d_2, d_3.

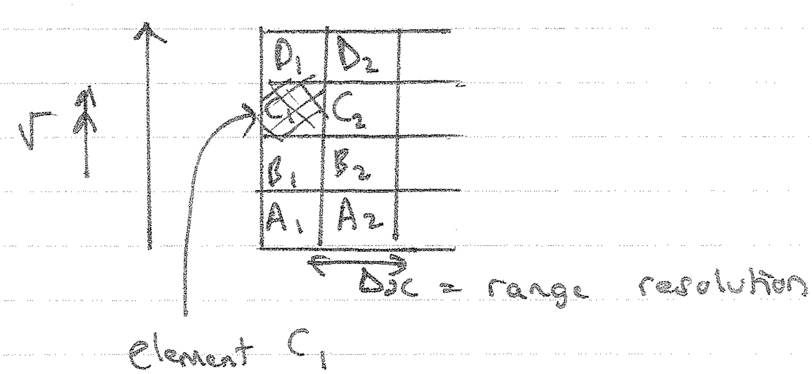
Pulse T_b is emitted when the aircraft is in line with row B. Unfortuneately, the return at bin b_1 does not come from area element B_1 only. Because of the antenna beamwidth there are returns from elements $A_1, C_1, D_1, E_1 \ldots$.

A typical SAR processor is shown over.

The scene depicted in the processor corresponds to the moment when the processor outputs the ground return associated with ground element C_1.

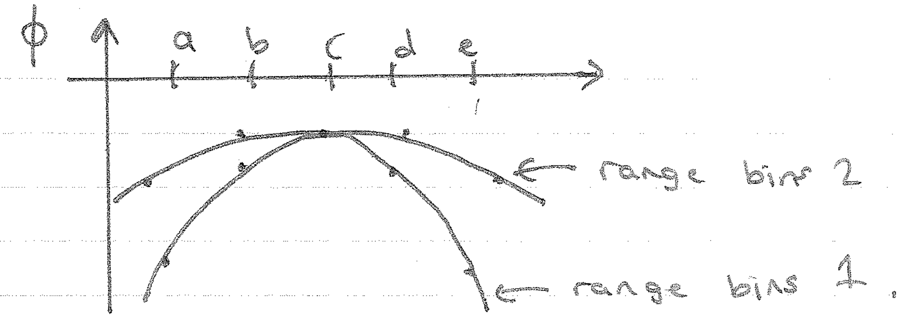


We note that the output corresponding to element C_1, ie,



Contains returns from neighboring elements, ie A_1, B_1, D_1 & E_1, corresponding to bins a_1, b_1, d_1 & e_1, in addition to bin c_1.

Each of the neighboring bins is suitably weighted to compensate for its associated phase shift. The phase shift weights are parabolic (quadratic), and a different for each range, ie



We note that the next range bin, eg 2, corresponding to $A_2, B_2, \ldots$ has a different phase compensation curve given its longer range i.e. Lower Doppler.

In summary, the processor basically performs two correlations:
range correlation by matched filter
& azimut correlation by multiplies ⊗

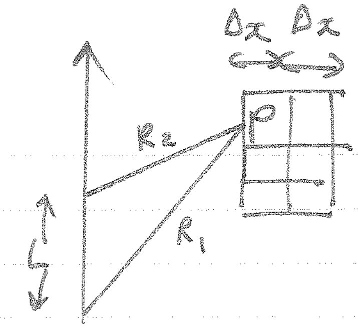
The two correlations could be done in the frequency domain using FFT & FFT^{-1}.

Range Migration

A major problem in SAR is the change in range to a given point on the ground as the SAR is moving along the observation section of its path. This problem is called range migration.

The processing described previously assumes a return from a point on the ground remains in the same range bin during the entire length of the synthetic aperture. Doppler processing is based on range change, yet we want the range change to remain within a fraction of one range resolution element.

In order to remain in the same range bin the range change has to be smaller than the range



resolution element.

Thus the total range change, ie

$$\Delta R = R_1 - R_2 < \Delta x$$

From our previous expression

$$R = R_0 + \frac{V^2 t^2}{2R_0} - \ldots \ldots \qquad \theta_0 = 90^\circ$$

we find $\quad R_{MIN} = R(t=0) = R_0$

$$R_{MAX} = R(t = T/2) = R_0 + \frac{V^2 (T/2)^2}{2R_0}$$

$$\therefore \Delta R = \frac{(VT)^2}{8R_0} = \frac{L^2}{8R_0}$$

Typical values for airborne SAR : $L = 400m$ & $R_0 = 20km$ yielding $\Delta R = 1m$.

For spaceborne SAR $L = 13km$ & $R_0 = 850km$ $\therefore \Delta R = 25m$.

$\therefore$ If range resolution is 10m in both cases, range will clearly be a problem in spaceborne SAR.

Other complex SAR issues include the Earths rotation for spaceborne SAR, and the issue of moving target in any SAR.