Please use complete sentences. Avoid using abbreviations. Use a math symbol editor like Latex. Please, no stuff like <=.

W1. Write a short paragraph comparing and contrasting the method of undetermined coefficients and variation of parameters. How are they similar, how are they different? If you had your choice, which method would you use?

W2. Now consider the driven differential equation:

$$my''(t) + cy'(t) + ky(t) = mg + \sqrt{t}.$$

Why would the method of undetermined coefficients be difficult to use for this problem?

1. The method of undetermined coefficients (MUC) is a very elegant method to find a particular solution for a non-homogenous ODE with *constant* coefficients: $a_2 y'' + a_1 y' + a_0 y = g(x)$. Once the solution to the associated homogenous equation is found, we "guess" a solution that is a polynomial of the independent variable (*x* or *t*) multiplied by the homogenous solution. By substituting this guess into the non-homogenous equation we can derive the polynomial that will satisfy the equation, by equating the coefficients of like-terms on both sides of the equation. Since the uniqueness and existence theorem states that for an ODE with well defined initial condition exists a unique solution, the particular solution we derived added to the homogenous solution is sufficient.
2. The method of variation of parameters (VOP) is slightly different. Here we look for a solution to the non-homogenous equation $y'' + p(x)y' + q(x)y = g(x)$. After we solve the homogenous equation we obtain two linearly independent homogenous solutions we denote as $y_{1h}$ and $y_{2h}$ and we "guess" a solution in the form: $u_1(x)y_{1h} + u_2(x)y_{2h}$. Substituting this back into the non-homogenous equation we get a system of first order differential equations for $u_1(x)$ and $u_2(x)$ that are once solved, complete the solutions.

The similarities between the methods is in the "guessing" approach in which we build a particular solution based on the homogenous solution, and substituting it back into the original equation to obtain a valid particular solution.
The MUC is much easier than VOP due to the fact that A. We are dealing with an equation with constant coefficients, thus the homogenous solution is readily available and does not require integration, while in VOP method the homogenous solution is not always trivial. B. Once we found the homogenous solution, the MUC does not require

additional work to solve more differential equations (hence more integrations) which is the case in the VOP.
However, the main two disadvantages of MUC is in the fact that it is good only for a narrow class of ODE's (namely – constant coefficients) and it is not simple (almost impossible) unless the non-homogenous term $g(x)$ is a function of the form: $P(x)e^{\alpha x}\cos(\beta x)$ or $Q(x)e^{\alpha x}\cos(\beta x)$ where $P(x)$ and $Q(x)$ are polynomials of $x$.

In contrast, the VOP method is applicable to a more general type of equation, but it requires more computational prowess, and it doesn't guarantee an explicit solution, since we still need to solve differential equations that might be solved only implicitly.

W2.
The equation of the form

$$my'' + cy' + ky = mg - \sqrt{t}$$

Is far from ideal to solve using MUC. While the equation satisfies the condition of constant coefficients, it does not satisfy the requirement that the non-homogenous term is of a special form $P(x)e^{\alpha x}\cos(\beta x)$ or $Q(x)e^{\alpha x}\cos(\beta x)$.