

# Idha Art

Note Title

03/05/2009

## 1) Arithmetics

The sequence is:  $\{0.4, 0.7, 1, 1.1, 1.4, 1.7, 2.0\}$ .

1.(a) Neither; no common difference or common ratio.

(b), Has a common difference b/w successive terms.

(c) Has a common ratio b/w successive terms.

2).  $t_1 = 2, t_5 = 162$ , what is:  $t_1, t_2, t_3 = ?$

Assume sequence is geometric! ← because  $t_5 > t_1$ ;

Given:  $t_n = ar^{n-1}$ ;

$$t_1 = ar^0 = \boxed{a} = 2. \checkmark$$

$$\textcircled{2} t_5 = ar^4 = 2r^4$$

$$\Rightarrow 162 = 2r^4 \Rightarrow 81 = r^4 \Rightarrow \boxed{r = 3}$$

∴ The sequence is:

$$\boxed{t_n = 2(3)^{n-1}}$$

$$\Rightarrow t_1 = 2, t_2 = 2(?) = 6$$

$$t_3 = 2(3)^2 = 18.$$



(b) The sequence is  $\{2, 8, 14, \dots\}$ .

Arithmetic sequence:  $d = 6$

Want to know for what  $n$ ,  $t_n = 128$ .

$$d = 6, \quad t_n = a + (n-1)d$$

$$a = 2, \quad d = 6$$

$$\begin{aligned} \therefore t_n &= 2 + 6(n-1) \\ &= 2 + 6n - 6 \\ &= 6n - 4 = 2(3n - 2). \end{aligned}$$

$$128 = 2(3n-2) \Rightarrow 64 = 3n-2 \Rightarrow n = 22$$

$\therefore$  The 22<sup>nd</sup> term is equal to 128.



(c)  $t_1 = 2$ ,  $t_{20} = 40$ . What is  $\{t_1, \underline{t_2}, t_3, t_4\}$ .

Since  $t_{20}$  seems to grow in succession with  $t_1$ , assume the sequence is arithmetic ...

$$t_1 = 2, \quad t_{20} = 40;$$

$$\textcircled{1} \quad t_n = a + (n-1)d$$

$$\Rightarrow t_1 = 2 = a \quad \checkmark.$$

$$\textcircled{2} \quad t_{20} = a + 19d.$$

Since  $a = \underline{2}$ :  $t_{20} = 2 + 19d$ .

$$\Rightarrow (40) = 2 + 19d.$$

$$\Rightarrow \frac{38}{19} = d. \Rightarrow \boxed{d = 2}$$

$$\therefore t_n = 2 + (n-1)2$$

$$= 2 + 2n - 2$$

$$= 2n$$

$$\therefore \boxed{t_1 = 2, \quad t_2 = 4, \quad t_3 = 6, \quad t_4 = 8}$$



$$(d) \quad t_6 = ?; \quad t_{10} = ?; \quad \text{what is } \underline{t_1} = ?$$

Assume sequence is arithmetic.  $\checkmark$

$$t_n = a + (n-1)d.$$

$$\textcircled{1} \quad t_6 = a + 5d = ?$$

$$\textcircled{2} \quad t_{10} = a + 9d = 19.$$

$$\begin{array}{l} \text{so } a + 5d = 7 \quad (1) \\ a + 9d = 19. \quad (2) \end{array}$$

Subtracting:  $4d = 12 \Rightarrow d = 3$

Subbing into (1):  $a + 5(3) = 7$   
 $\Rightarrow a = -8$

$$\begin{array}{l} \text{so } t_n = -8 + (n-1)3 \\ = -8 + 3n - 3 \\ = 3n - 11 \end{array}$$

∴  $t_n = 3n - 11$

And:  $t_1 = 3(1) - 11 = -8$



### (3) FRACTAL Thro' Question:

The number of line segments are:

$$\{0, 2, 4, 8, 16\}.$$

Sequence is geometric:

$$t_n = ar^{n-1}, \text{ het } a = 2; r = 2.$$

$$\therefore t_n = 2 \cdot (2)^{n-1}.$$

$$\therefore t_{20} = 2 \cdot (2)^{19} = \boxed{1048576}$$

This is actually  $t_{21}$ , since we ignored  $t_1$ :

$$\therefore t_{21} = 1048576$$

$$\therefore t_{19} \text{ gives } t_{20}: \quad t_{19} = 2 \cdot (2)^{19-1} \\ = \boxed{524288}$$

~~524288~~

$$(4) \quad \boxed{t_1 = -3 = a} \Rightarrow t_2 = 2 + t_1 \\ t_3 = 2 + t_2.$$

$$(a) \quad t_1 = -3$$

↓

$$t_2 = 2 + (-3) = -1. \quad \therefore \text{the first 4 terms are:} \\ t_3 = 2 + (-1) = 1 \quad \{ -3, -1, 1, 3 \} \\ t_4 = 2 + (1) = 3. \quad \underline{\hspace{2cm}}$$

$$(b) \quad d = 2, \quad a = -3:$$

$$\therefore \boxed{t_n} = -3 + (n-1)2 \\ = -3 + 2n - 2 \\ = \boxed{2n - 5}$$

(c) Recursive formula:  $t_1 = -3$

$$\therefore \boxed{\underline{t_n = 2 + t_{n-1}}}$$

(d)  $t_{50} = 2(50) - 5 = \boxed{95}$

(e) Used the arithmetic formula; because the recursive formula would want the first 49 terms!!.

~~====~~

(f) Sequence: Arithmetic:  $\{a, x, b\}$

Given that:  $x = 7$ ,  $a^2 + b^2 = 148$ .

But:  $x$  is an arithmetic mean:  $= 7$ .

$$\text{So: } \frac{a+b}{2} = 7.$$

$$\textcircled{1} \quad a+b = 14 \Rightarrow b = 14-a.$$

$$\Rightarrow b^2 = (14-a)^2.$$

∴ Substituting into:  $a^2 + b^2 = 148$ :

$$a^2 + (14-a)^2 = 148$$

$$\Rightarrow a^2 + 196 - 28a + a^2 = 148$$

$$\Rightarrow 2a^2 - 28a + 48 = 0$$

$$\Rightarrow 2(a^2 - 14a + 24) = 0$$

$$\Rightarrow a^2 - 14a + 24 = 0$$

$$\Rightarrow (a - 2)(a - 12) = 0$$

$$\Rightarrow \boxed{a = 2, a = 12}$$

Since:  $b^2 = (14 - a)^2$

So:  $b_1^2 = (12)^2 \Rightarrow \boxed{b_1 = 12}$

or:  $b_2^2 = 2^2 \Rightarrow \boxed{b_2 = 2}$

So, the sequence is either  $\{2, 3, 12\}$  or  $\{12, 7, 2\}$ .

Cone sequence just reversed; This happens because of the condition:  $\underline{\underline{a^2 + b^2 = 148}}$



(6)  $t_1 = a = 18500, d = \underline{\underline{1500.00}}$

(a) This is an arithmetic sequence with  $n = \text{the } n^{\text{th}} \text{ year.}$

$$t_n = a + (n-1)d$$

$$\Rightarrow t_{12} = a + (11)d$$
$$= 18500 + (11)(1500)$$
$$= \$35000.00$$

∴ The minimum salary in 12 years will be:  
 $\$35000.00$

(b) We want the sum of the 1<sup>st</sup> 12 years of the  
= arithmetic series

$$S_n = \frac{n}{2} [2(a) + (n-1)d]$$

$$\Rightarrow S_{12} = \frac{12}{2} [2(18500) + 11(1500)]$$

$$= 6 [53500] = \boxed{\$321000.00}$$

~~XXXX~~

$t_1, t_2, t_3, t_4, t_5, t_6$

$$(7) \quad a = \$275.00; \quad t_1 = \$275.00;$$

$$t_2 = (1.12)t_1, \quad t_3 = (1.12)t_2.$$

Clearly, a geometric sequence with :  $a = 275$ ,  
 $r = 1.12$ ;

$$\therefore t_n = ar^{n-1} \\ = (275)(1.12)^{n-1}$$

$$\therefore t_6 = (275)(1.12)^5 \\ = \underline{\underline{\$484.64}}$$

So after the 6<sup>th</sup> year, the stamp has a minimum value of at least  $\underline{\underline{\$484.64}}$

~~✓~~

(8) the sequence is  $\{1, 6, 7, 6, 3, 6, 11\}$ .

Next 3 tens are  $\boxed{6, 1, 6}$ . You first do sum  
and then the difference:

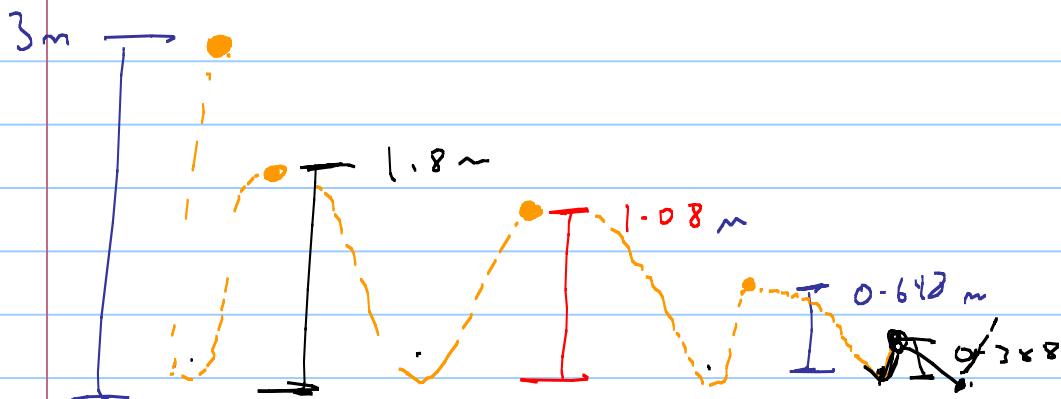
$$1+6=7,$$

$$6-1=5,$$

$$5+6=11;$$

$6-5=1$ ; And in the middle we put 6, that  
is why every other term is 6  $\text{☒}$

(9) BASIC TRICK: OK, obviously geometric:



From the diagram, the sequence is:

$$\{3, 1.8, 1.08, 0.648, 0.388\}.$$

$$a = 3, r = 0.60.$$

$$\therefore \boxed{h_n = 3 (0.60)^{n-1}} = \text{height}$$

∴, the total distance travelled when it hits the ground for the 5<sup>th</sup> time is:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{3(0.60^5 - 1)}{0.60 - 1} = \boxed{6.92 \text{ m}}$$

∴ The total distance travelled by the ball when it hits the ground for the 5<sup>th</sup> time is:  
 $\approx 7.15 \text{ m.}$

~~$\sum n^3 - n^2 + 2n$~~

$$\begin{aligned} & [n^3 - n^2 + 2n]^5 \\ & (5)^3 - 5^2 + 2(5) = 1^3 + 1^2 - 2 \end{aligned}$$

(D)  $f(n) = \underline{\underline{3n^2 - 2n + 2}}, \quad n \in \mathbb{N};$

First five terms:  $n=1, 2, 3, 4, 5$

(1)  $f = \{ \underline{\underline{3}}, \underline{\underline{10}}, \underline{\underline{23}}, \underline{\underline{42}}, \underline{\underline{67}} \};$

Sketch? Check your e-mail!

$$(b) S_5 = 145$$

$$(ii) S_{100} = 1005150;$$

$$(iii) S_{300} = 27,045,450$$

~~~~~~~~~  $\tilde{\text{Bawas Q:}}$

Geometric Series: 3 terms;

Let these terms be:  $t_1, t_2, t_3$ .

$$S_0: \textcircled{1} \quad S_3 = 42;$$

$$\textcircled{2} \quad t_3 = 3 \cdot 2 (t_1 + t_2)$$

$$t_n = a r^{n-1}$$

$$t_3 = a r^2 = 3 \cdot 2 (t_1 + t_2)$$

$$S_0: t_1 r^2 = 3 \cdot 2 (t_1 + t_2)$$

$$\text{And: } S_3 = \frac{t_1 (r^3 - 1)}{r - 1} = 42.$$

50, -70, 32

$$\underline{\text{So:}} \quad t_2 = \frac{t_1(r^3 - 1)}{r - 1}$$

$$\underline{\text{So:}} \quad \boxed{t_1 = \frac{42(r-1)}{(r^3 - 1)}}$$

$$t_2 = t_1 r = 42 \frac{r(r-1)}{r^3 - 1}$$

$$t_3 = t_1 r^2 = 42 \frac{r^2(r-1)}{r^3 - 1}$$

So, the three times are:

$$\left\{ \frac{42(r-1)}{(r^3 - 1)}, \frac{42r(r-1)}{(r^3 - 1)}, \frac{42r^2(r-1)}{(r^3 - 1)} \right\}.$$

What is r?

$$\frac{42r^2(r-1)}{r^3 - 1} = 3.2 \left( \frac{42(r-1)}{r^3 - 1} + \frac{42r(r-1)}{r^3 - 1} \right).$$

$$= 3.2 \left( \underbrace{\frac{42(r-1) + 42r(r-1)}{(r^3 - 1)}} \right)$$

$$\Rightarrow \frac{42r^2(r-1)}{r^3-1} = 3 \cdot 2 \left( \frac{+2r-42+42r^2-42}{r^3-1} \right)$$

$$\Rightarrow \frac{42r^2(r-1)}{r^3-1} = 3 \cdot 2 \left( \frac{42r^2-42}{r^3-1} \right) .$$

$$\Rightarrow 42r^2(r-1) = 3 \cdot 2 (42r^2 - 42) .$$

$$\Rightarrow 42r^2(r-1) = 3 \cdot 2 (42(r^2-1)) .$$

$$\Rightarrow r^2(r-1) - 3 \cdot 2r^2 + 3 \cdot 2 = 0$$

$$\Rightarrow r^3 - r^2 - 3 \cdot 2r^2 + 3 \cdot 2 = 0$$

$$\Rightarrow r^3 - 4 \cdot 2r^2 + 3 \cdot 2 = 0$$

$$\Rightarrow r = \frac{-4}{5}, 1, 4 \quad \begin{matrix} \leftarrow \text{don't match} \\ \text{third condition} \\ \text{of } t_3 = 3 \cdot 2(t_1 + t_2) \end{matrix}$$

✓      ↗      ↗      ↗

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not reasonable,

since gives  
0 in the  
sequence.

$$r = -\frac{4}{5}$$

Want to  
solve  
which  
you've  
done + will  
do, say you  
want to do!  
=

Subbing into the sequence.

$$\left\{ \frac{42(r-1)}{(r^3-1)}, \frac{42r(r-1)}{(r^3-1)}, \frac{42r^2(r-1)}{(r^3-1)} \right\}.$$

the three numbers:

=

$$\left\{ 50, -40, 32 \right\}.$$

