

PROBLEM:

Solve the 2D heat equation

$$\phi_t = a^2[\phi_{xx} + \phi_{yy}] \quad 0 \leq x \leq x_0 \quad y > 0$$

with initial conditions

$$\phi(x, y, 0) = 1$$

and boundary conditions

$$\phi_y(x, 0, t) = 0$$

$$\phi(0, y, t) = 0$$

$$\phi(x_0, y, t) = 0$$

along with the boundedness condition

$$\phi(x, y, t) \text{ bounded as } y \rightarrow \infty$$

Justify the convergence of any series solutions.