1. Let *f* be a function defined on **R** and, for each natural number n, define the function f_n by $f_n(x) = f\left(x + \frac{1}{n}\right)$. In each of the following, decide whether or not you believe the statement is true. If the statement is true, give a complete and careful proof. If the statement is false, give an explicit counterexample. In the latter case, be sure to prove that your counterexample has the properties that you claim for it.

a) If *f* is uniformly continuous, then the sequence $\{f_n\}$ converges uniformly on **R**.

b) If *f* is continuous, then the sequence $\{f_n\}$ converges uniformly on **R**.

c) If *f* is continuous, then the sequence $\{f_n\}$ converges pointwise on **R**.

d) The sequence $\{f_n\}$ converges pointwise on **R** (without any additional assumptions on *f*.)