3. What physical conditions lead electrons in a metal to form a degenerate Fermi gas?

Show that the density of states for an ideal two-dimensional Fermi gas of particles with spin $\frac{1}{2}$ and mass m that occupies area A is:

$$g(E) = \frac{A}{2\pi}\left(\frac{2m}{\hbar^2}\right).$$

Obtain a relationship between the number of electrons per unit area and the Fermi energy E_F at 0 K.

Show that the average electron energy for the two-dimensional gas at 0 K is $E_\mathrm{F}/2$.

Obtain the temperature dependence, at low temperatures, of the heat capacity of unit area of the Fermi gas. Explain why the magnitude of the heat capacity is much smaller than the value predicted for a classical two-dimensional gas. Obtain the temperature dependence, at low temperatures, of the contribution of the lattice vibrations to the heat capacity of a very thin film of a metal.