

Consider the following heat equation with inhomogeneous boundary conditions defined on the interval $0 < x < l$:

$$\phi_t = a^2 \phi_{xx} \quad \phi(0, t) = 1, \quad \phi(l, t) = 2 \quad \phi(x, 0) = 0.$$

Solve this problem by converting it into a problem with homogeneous boundary conditions. Show that as $t \rightarrow \infty$ the solution approaches the equilibrium solution.