

1. Show that a bounded set  $\{x \in \mathbb{R} \mid |x| < M\}$ ,  
has  $\sup_{x \in S} x$ .

2. The space of all real numbers is the  
completion of all rational numbers.  
( $\mathbb{R}^1 = \mathbb{Q}^*$ ).

Hint from my professors.

Let  $Q_{10}$  be the space of finite decimal fractions.

$$Q_{10} = \left\{ \frac{m}{10^N} \mid m \in \mathbb{Z} \right\} \iff f = \sum_{n=M}^N a_n 10^{-n}$$

$$a_M: \dots a_1 \cdot a_1 a_2 a_3 \dots a_N$$

$$a_{M+1}: \quad \vdots \quad \vdots$$

$$\cdot a_1 a_2 a_3 a_4 a_N a_{N+1}$$

\*

Q3. The set of all points  $(x = x_1, x_2, \dots, x_n)$  with  
rational coordinates is dense in  $\mathbb{R}^n$ .  
(Please see the example 3 on page 48 in my  
reference-6 pdf file.)

$$[Q] = \mathbb{R}$$

\*Q4. P. 49 in my reference-6 pdf.

Example 7. The end of the fifth line says that  
"the distance b/w any two points of  $E$   
equals 1." ~~So why~~ why is it equal to 1?

Please explain.

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Q5. Given set  $M$ , Let  $M \subset B$ .  $B$  is closed.  
Show that  $B \supset \bar{M}$ . ( $\bar{M}$  is the set of  
all contact points  
of  $M$ )