MATH 137 - ASSIGNMENT 8

Submit the following problems by 8:20 a.m. **March 20** in the drop boxes opposite to MC 4066. All solutions must be **clearly stated** and **fully justified**. Use the format in the file Math 135 and Math 137 Assignment Templates.

1. Find the limit. You may use any of the rules that you have learned so far. For more problems to practice, refer to the list of optional problems at the end of this assignment.

   (a) $\lim_{x\to 1} \dfrac{x^a - ax + a - 1}{x^b - bx + b - 1}$, where a, b are real numbers such that $b \neq 0, 1$.

   (b) $\lim_{x\to 0} \dfrac{e^x(x - \sin x)}{(x - \tan x)(x+1)}$ (c) $\lim_{x\to -\infty} \dfrac{e^x - 2}{e^x + 1}$ (d) $\lim_{x\to 0}(4x+1)^{(1/\sin x)}$

2. Use the guidelines in Section 4.5 to sketch the graph of the given function. Remember to label all intersects, local extremes, inflection points and asymptotes. For more problems to practice, refer to the list of optional problems at the end of this assignment.

   (a) $\dfrac{1}{x^2 - 4}$ (b) $x^2 \ln x$ (c) xe^{-x}

3. Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.

4. A cylindrical can with a top is made to hold V cm^3 of liquid. Find the dimensions (the radius and the height) that will minimize the cost of the metal to make the can.

5. A piece of wire L meters long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) a minimum?

6. Section 4.7 Problem 68 on page 332. (You may need the formulas: $\sin(2\theta) = 2\sin\theta\cos\theta$ and $\cos(2\theta) = 2\cos^2\theta - 1$.)

7. Suppose that f is concave upward and differentiable on (a, b). Show that f' is increasing. (Hint: use the definition of concavity.)

8. Suppose that f is concave upward and differentiable on (a, b). Let $x < y$ in $[a, b]$ and $\lambda \in (0, 1)$, show that

$$f(\lambda x + (1-\lambda)y) < \lambda f(x) + (1-\lambda)f(y).$$

   What is the analogous inequality for a differentiable concave downward function? (Remark: see also Problem 14 in Assignment 1).

9. Prove that for all x in $(0, \frac{\pi}{2})$, we have $\frac{2}{\pi}x < \sin(x)$.

10. Section 4.3 Problem 1 on page 295.

11. Given that the graph of f passes through the point $(1, 6)$ and that the slope of its tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

12. Find the function f, given that $f''(t) = 2e^t + 3\sin t$, $f(0) = 0$, $f(\pi) = 0$.

13. Find the function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

The following problems are suggested for use in preparing for your assignments and/or for review for tests and examinations. **Do not submit these problems**.

Section 4.3 - 9, 11, 15, 35, 39, 45, 47, 69

Section 4.4 - 5, 7, 9, 11, 15, 17, 19, 21, 27, 29, 31, 43, 47, 49, 53, 55, 57, 60, 61, 69, 70, 71, 79

Section 4.5 - 1, 7, 9, 13, 23, 31, 37, 43, 48, 49

Section 4.7 - 11, 12, 17, 31, 33, 46