Section 4.2.4). By conditioning on N_u, we find

$$E(T \mid U = u) = \sum_{n=0}^{\infty} E(T \mid U = u, N_u = n)P(N_u = n)$$

$$= \sum_{n=0}^{\infty}(u + nd)P(N_u = n)$$

$$= u\sum_{n=0}^{\infty} P(N_u = n) + d\sum_{n=0}^{\infty} nP(N_u = n),$$

yielding $E(T \mid U = u) = u + d\lambda u$. Hence,

$$E(T) = \int_a^b (u + d\lambda u)f(u)\,du = (1 + \lambda d)\int_a^b uf(u)\,du$$

$$= (1 + \lambda d)E(U).$$

Since $E(U) = (a + b)/2$, the expected value of the time until completion of the unloading of the ship is equal to $(1 + \lambda d)(a + b)/2$.

Problem 13.5 Consider the casino game Red Dog from Problem 3.27 again. Suppose that the initial stake of the player is \$10. What are the expected values of the total amount staked and the payout in any given play? Use the law of conditional expectations to find these expected values.

Problem 13.6 A replenishment order is placed to raise the stock level of a given product. The current stock level is s units. The lead time of the replenishment order is a continuous random variable having an exponential distribution with a mean of $1/\mu$ days. Customer demand for the product occurs according to a Poisson process with an average demand of λ units per day. Each customer asks for one unit of the product. What is the probability of a shortage occurring during the replenishment lead time and what is the expected value of the total shortage?

Problem 13.7 A fair coin is tossed no more than n times, where n is fixed in advance. You stop the coin-toss experiment as soon as the proportion of heads exceeds $\frac{1}{2}$ or as soon as n tosses are done, whichever occurs first. Use the law of conditional expectations to calculate, for $n = 5, 10, 25$, and 50, the expected value of the proportion of heads at the moment the coin-toss experiment is stopped. *Hint*: define the random variable $X_k(i)$ as the proportion of heads upon stopping given that k tosses are still possible and heads turned up i times so far. Set up a recursion equation for $E[X_k(i)]$.