

If $G = (V, U, E)$ is a bipartite graph satisfying $|V| = |U| = n$ and $|N(S)| \geq |S| - d$ for every $S \subset V$ and some fixed integer d . Here as always, $N(S) \subset U$ refers to the set of neighbors of the elements of S , i.e., $N(S)$ is the set of all those vertices adjacent to at least one member of S . Prove that the graph G has a matching of size $n - d$. (Hint: Try to use Hall's theorem on a suitably modified graph.)