

“8”

Limits

In Exercises 1 to 8, find the limit of each sequence that converges; if the sequence diverges, explain why.

$$8. z_n = n \left\{ 1 - \cos\left(\frac{\theta}{n}\right) - i \sin\left(\frac{\theta}{n}\right) \right\}, \quad \theta \text{ fixed}$$

“32” n “36”

30. Suppose that $\sum_{n=1}^{\infty} a_n$ converges. Show that $\sum_{n=1}^{\infty} n a_n$ diverges.

In Exercises 31 to 39, determine whether the given infinite series converges or diverges.

$$31. \sum_{n=1}^{\infty} \left(\frac{1+2i}{\sqrt{6}} \right)^n$$

$$32. \sum_{n=1}^{\infty} n \left(\frac{1}{2i} \right)^n$$

$$33. \sum_{n=1}^{\infty} \left(\frac{2+i}{\sqrt{5}} \right)^n$$

$$34. \sum_{n=1}^{\infty} \frac{1}{2+i^n}$$

$$35. \sum_{n=2}^{\infty} n(n-1)\beta^{n-2}, \quad |\beta| < 1$$

$$36. \sum_{n=1}^{\infty} \frac{1}{n^2 + i^n}$$

$$37. \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{1+i}{\sqrt{2}} \right)^n$$

$$38. \sum_{n=1}^{\infty} \frac{k^2 + i}{n}$$

“14”

Find the values(s) of the given expression in Exercises 1 to 14.

$$14. \exp \left[\pi \left(\frac{i+1}{\sqrt{2}} \right)^4 \right]$$

“14”

14. Let D_1 and D_2 be domains with a nonempty intersection. Show that $D_1 \cup D_2$ is a domain.