

Permutation groups G_1 and G_2 acting on the sets S_1 and S_2 are called *permutation isomorphic* if there exists an isomorphism $\theta : G_1 \rightarrow G_2$ and a bijection $\phi : S_1 \rightarrow S_2$ such that $(\theta x)(\phi s) = \phi(xs) \forall x \in G_1$ and $s \in S_1$. In other words, the following diagram commutes:

$$\begin{array}{ccc} S_1 & \xrightarrow{x} & S_1 \\ \phi \downarrow & & \downarrow \phi \\ S_2 & \xrightarrow{\theta x} & S_2 \end{array}$$

Define two group actions of a group G on itself as follows:

- (i) the action of $x \in G$ is left multiplication by x ;
- (ii) the action of $x \in G$ is right multiplication by x^{-1}

Show that the two actions are *permutation isomorphic*.