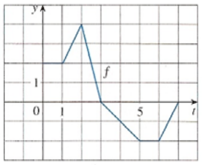
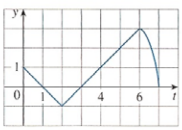
5.4 EXERCISES

1. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.
   (a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.
   (b) On what interval is g increasing?
   (c) Where does g have a maximum value?
   (d) Sketch a rough graph of g.



2. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.
   (a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6.
   (b) Estimate $g(7)$.
   (c) Where does g have a maximum value? Where does it have a minimum value?
   (d) Sketch a rough graph of g.



3–4 ▪ Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

3. $g(x) = \int_0^x (1 + t^2)\,dt$ 4. $g(x) = \int_0^x \left(1 + \sqrt{t}\right)dt$

5–14 ▪ Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

5. $g(x) = \int_0^x \sqrt{1 + 2t}\,dt$ 6. $g(x) = \int_1^x \ln t\,dt$

7. $g(y) = \int_2^y t^2 \sin t\,dt$

8. $F(x) = \int_x^{10} \tan\theta\,d\theta$

$$\left[\textit{Hint: } \int_x^{10} \tan\theta\,d\theta = -\int_{10}^x \tan\theta\,d\theta\right]$$

9. $h(x) = \int_2^{1/x} \arctan t\,dt$ 10. $h(x) = \int_0^{x^2} \sqrt{1 + r^3}\,dr$

11. $y = \int_3^{\sqrt{x}} \dfrac{\cos t}{t}\,dt$ 12. $y = \int_{e^x}^0 \sin^3 t\,dt$

13. $g(x) = \int_{2x}^{3x} \dfrac{u^2 - 1}{u^2 + 1}\,du$

$$\left[\textit{Hint: } \int_{2x}^{3x} f(u)\,du = \int_{2x}^0 f(u)\,du + \int_0^{3x} f(u)\,du\right]$$

14. $y = \int_{\sin x}^{\cos x} (1 + v^2)^{10}\,dv$

15–18 ▪ Find the average value of the function on the given interval.

15. $f(x) = x^2$, $[-1, 1]$ 16. $f(x) = 1/x$, $[1, 4]$

17. $g(x) = \cos x$, $[0, \pi/2]$

18. $f(\theta) = \sec\theta \tan\theta$, $[0, \pi/4]$

19–20 ▪
(a) Find the average value of f on the given interval.
(b) Find c such that $f_{\text{ave}} = f(c)$.
(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

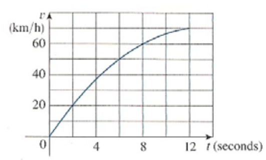
19. $f(x) = (x - 3)^2$, $[2, 5]$

20. $f(x) = \sqrt{x}$, $[0, 4]$

21. The table gives values of a continuous function. Use the Midpoint Rule to estimate the average value of f on $[20, 50]$.

x	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

22. The velocity graph of an accelerating car is shown.
   (a) Estimate the average velocity of the car during the first 12 seconds.
   (b) At what time was the instantaneous velocity equal to the average velocity?



24. Find the interval on which the curve

$$y = \int_0^x \frac{1}{1 + t + t^2}\, dt$$

is concave upward.

25–26 ▪ Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.

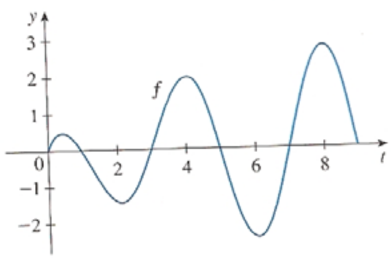
(a) At what values of x do the local maximum and minimum values of g occur?

(b) Where does g attain its absolute maximum value?

(c) On what intervals is g concave downward?

(d) Sketch the graph of g.

25.



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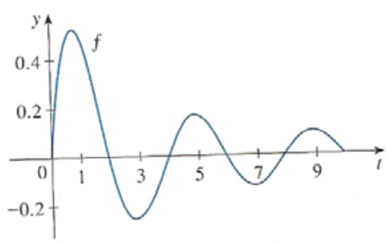

27. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\, dx = 17$, what is the value of $f(4)$?

28. The **error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

is used in probability, statistics, and engineering.

(a) Show that $\int_a^b e^{-t^2}\, dt = \frac{1}{2}\sqrt{\pi}\,[\operatorname{erf}(b) - \operatorname{erf}(a)]$.

(b) Show that the function $y = e^{x^2}\operatorname{erf}(x)$ satisfies the differential equation $y' = 2xy + 2/\sqrt{\pi}$.

29. The Fresnel function S was defined in Example 4 and graphed in Figures 7 and 8.

(a) At what values of x does this function have local maximum values?

$$\int_0^x \sin(\pi t^2/2)\, dt = 0.2$$

CAS **30.** The **sine integral function**

$$\operatorname{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

(a) Draw the graph of Si.

(b) At what values of x does this function have local maximum values?

(c) Find the coordinates of the first inflection point to the right of the origin.

(d) Does this function have horizontal asymptotes?

(e) Solve the following equation correct to one decimal place:

$$\int_0^x \frac{\sin t}{t}\, dt = 1$$

31. Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$$

for all $x > 0$.

32. A high-tech company purchases a new computing system whose initial value is V. The system will depreciate at the rate $f = f(t)$ and will accumulate maintenance costs at the rate $g = g(t)$, where t is the time measured in months. The company wants to determine the optimal time to replace the system.

(a) Let

$$C(t) = \frac{1}{t}\int_0^t [f(s) + g(s)]\, ds$$

Show that the critical numbers of C occur at the numbers t where $C(t) = f(t) + g(t)$.

(b) Suppose that

$$f(t) = \begin{cases} \dfrac{V}{15} - \dfrac{V}{450}t & \text{if } 0 < t \leqslant 30 \\ 0 & \text{if } t > 30 \end{cases}$$

and $$g(t) = \frac{Vt^2}{12{,}900} \qquad t > 0$$