Homework 3
September 8, 2005
due on Thursday, Sept. 15

1. Suppose that the sample space $S = \{1, 2, 3, ...\}$. Let $p_k = \Pr(\{k\})$ for $k \in S$. In each of the following cases, compute c. (a) Suppose that $p_k = c(5/6)^k$ for $k \in S$; (b) Suppose that $p_k = c(5/6)^k/(k)!$ for $k \in S$.

2. Suppose that the sample space is $S = [0, \infty)$. Let $B_t = [t, \infty)$ for any $t \geq 0$. Suppose that $\Pr(B_t) = ce^{-6t}$ for $t \geq 0$. Compute (a) c, (b) $\Pr(B_2)$, and (c) $\Pr([1, 2))$.

3. Suppose we are dealt 6 cards from a standard well-shuffled deck. What is the probability that there are (a) a six card flush? (b) 4 of one kind and 2 of another? (c) two triples? (d) 3 pairs? (e) 2 pairs? (f) 1 pair? (g) at least one pair? You may leave your answer in terms of $\binom{n}{k}$.

4. Suppose that we have 30 different pairs of socks (60 socks in total) in a laundry basket. By different,we mean different colors or patterns so that each sock has a unique mate. Suppose that we select 10 socks at random from the basket. What is the $\Pr(A_k)$ for $k = 0, \ldots, 6$ where A_k is the event that we have exactly k pairs among the 10 randomly selected socks? (Hint: To start, it might help to figure out which of those events have probability zero.)

5. Problem 1.5.3 on p. 54.

6. For the birthday problem let p_k be the probability that k people all have different birthdays. In class, we derived an expression for p_k assuming 366 days in a year that are equally likely to be someone's birthday. We can express $p_{k+1} = p_k f_k$ where f_k is a certain factor that depended on k. (a) What is f_k? (Hint: since p_k is the probability that k people all have the distinct birthdays, we can think of f_k as the conditional probability that one more person entering the room will also have a distinct birthday given that the k already in the room all have distinct birthdays.

   We are interested in computing $1 - p_k$ for the number of people in the class. We are also interested in knowing when $1 - p_k$ is greater than $1/2$ since in these cases the bet is favorable for me.

   Set up an excel sheet as follows. In the first row, put the following headers: k, f_k, p_k, and $1 - p_k$. In the first column below k have the numbers from 1 up to 70. In the second column have the factor f_k corresponding to the number k in the same row in the first column. In the first row, put the (obvious?) value for p_1. The remainder of the p_k's will be computed using the recursion; i.e., multiplying two numbers from the row above. Lastly, compute the last column with the values of $1 - p_k$.

   (b) If there are 70 people in the class, what is the probability that I win the bet? (c) What is the smallest class size where the probability of me winning is greater than $1/2$?