

12. A star of mass M and radius R is moving with velocity v through a cloud of particles of density ρ . If all the particles that collide with the star are trapped by it, show that the mass of the star will increase at a rate

$$\frac{dM}{dt} = \pi\rho v \left(R^2 + \frac{2GMR}{v^2} \right).$$

Given that $M = 10^{31}$ kg and $R = 10^8$ km, find how the effective cross-sectional area compares with the geometric cross-section πR^2 for velocities of 1000 km s^{-1} , 100 km s^{-1} and 10 km s^{-1} .