EXAMPLE 6.10

This example presents a simplified version of calculations used by airlines when they overbook flights. They realize that a certain percentage of ticketed passengers will cancel at the last minute. Therefore, to avoid empty seats, they sell more tickets than there are seats, hoping that just about the right number of passengers show up. We will assume that the no-show rate is 5%. In binomial terms, we are assuming that each ticketed passenger, independently of the others, shows up with probability 0.95 and cancels with probability 0.05.

For a flight with 200 seats, the airline wants to find how sensitive various probabilities are to the number of tickets it issues. In particular, it wants to calculate (i) the probability that more than 205 passengers show up, (ii) the probability that more than 200 passengers show up, (iii) the probability that at least 195 seats will be filled, and (iv) the probability that at least 190 seats will be filled. The first two of these are "bad" events from the airline's perspective; they mean that some customers will be bumped from the flight. The last two events are "good" in the sense that the airline wants most of the seats to be occupied.

Solution

To solve the airline's problem, we use the BINOMDIST function and a data table. The solution appears in Figure 6.21. (See the file OVERBOOK.XLS.) We first enter a possible

FIGURE 6.21 Binomial Calculations for Overbooking Example

	A	B	C	D	E
1	**Airline Overbooking Problem**				
2					
3	Number of seats	200			
4	Probability of no-show	0.1		**Range names**	
5				NTickets: B6	
6	Number of tickets issued	215		PNoShow: B4	
7					
8	**Required probabilities**				
9		More than 205 show up	More than 200 show up	At least 195 seats filled	At least 190 seats filled
10		0.001	0.050	0.421	0.820
11					
12	**Data table showing sensitivity of probabilities to number of tickets issued**				
13	Number of tickets issued	More than 205 show up	More than 200 show up	At least 195 seats filled	At least 190 seats filled
14		0.001	0.050	0.421	0.820
15	206	0.000	0.000	0.012	0.171
16	209	0.000	0.001	0.064	0.384
17	212	0.000	0.009	0.201	0.628
18	215	0.001	0.050	0.421	0.820
19	218	0.013	0.166	0.659	0.931
20	221	0.064	0.370	0.839	0.978
21	224	0.194	0.607	0.939	0.995
22	227	0.406	0.802	0.981	0.999
23	230	0.639	0.920	0.995	1.000
24	233	0.822	0.974	0.999	1.000

number of tickets issued in cell B6 and, for this number, calculate the required probabilities in row 10. For example, the formulas in cells B10 and D10 are

=1-BINOMDIST(205,NTickets,1-PNoShow,1)

and

=1-BINOMDIST(194,NTickets,1-PNoShow,1)

Note how the wording "more than" is slightly different from "at least." The probability of more than 205 is 1.0 minus the probability of less than or equal to 205, whereas the probability of at least 195 is 1.0 minus the probability of less than or equal to 194. Also, note that we are treating a "success" as a passenger who shows up. Therefore, the third argument of each BINOMDIST function is 1.0 minus the no-show probability.

To see how sensitive these probabilities are to the number of tickets issued, we create a one-way data table at the bottom of the spreadsheet. It is *one-way* because there is only one *input*, the number tickets issued, even though there are four output probabilities tabulated. (To create the data table, list several possible numbers of tickets issued along the side in column A and transfer the probabilities from row 10 to row 14. That is, enter the formula =B10 in cell B14 and copy it across row 14. Then form a data table using the range A14:E24, no row input cell, and column input cell B6.)

The results are as expected. As the airline issues more tickets, there is a larger chance of having to bump passengers from the flight, but there is also a larger chance of filling most seats. In reality, the airline has to make a trade-off between these two, taking its various costs and revenues into account. ■