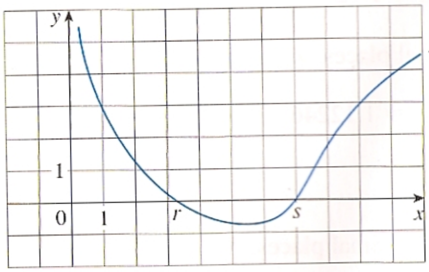
4.6 EXERCISES

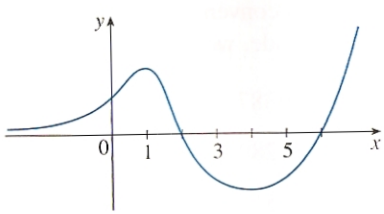
1. The figure shows the graph of a function f. Suppose that Newton's method is used to approximate the root r of the equation $f(x) = 0$ with initial approximation $x_1 = 1$.
(a) Draw the tangent lines that are used to find x_2 and x_3, and estimate the numerical values of x_2 and x_3.
(b) Would $x_1 = 5$ be a better first approximation? Explain.



2. Follow the instructions for Exercise 1(a) but use $x_1 = 9$ as the starting approximation for finding the root s.

3. Suppose the line $y = 5x - 4$ is tangent to the curve $y = f(x)$ when $x = 3$. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 3$, find the second approximation x_2.

4. For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown.
(a) $x_1 = 0$ (b) $x_1 = 1$ (c) $x_1 = 3$
(d) $x_1 = 4$ (e) $x_1 = 5$



5–6 ▪ Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation. (Give your answer to four decimal places.)

5. $x^3 + 2x - 4 = 0, \quad x_1 = 1$

6. $x^5 + 2 = 0, \quad x_1 = -1$

7. Use Newton's method with initial approximation $x_1 = -1$ to find x_2, the second approximation to the root of the equation $x^3 + x + 3 = 0$. Explain how the method works by first graphing the function and its tangent line at $(-1, 1)$.

8. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of the equation $x^4 - x - 1 = 0$. Explain how the method works by first graphing the function and its tangent line at $(1, -1)$.

9–10 ▪ Use Newton's method to approximate the given number correct to eight decimal places.

9. $\sqrt[3]{30}$ **10.** $\sqrt[7]{1000}$

11–12 ▪ Use Newton's method to approximate the indicated root of the equation correct to six decimal places.

11. The positive root of $\sin x = x^2$

12. The positive root of $2 \cos x = x^4$

13–20 ▪ Use Newton's method to find all the roots of the equation correct to eight decimal places. Start by drawing a graph to find initial approximations.

13. $x^5 - x^4 - 5x^3 - x^2 + 4x + 3 = 0$

14. $x^2(4 - x^2) = \dfrac{4}{x^2 + 1}$

15. $e^{-x} = 2 + x$ **16.** $\ln(4 - x^2) = x$

17. $x^2\sqrt{2 - x - x^2} = 1$ **18.** $3 \sin(x^2) = 2x$

19. $\tan^{-1}x = 1 - x$ **20.** $\tan x = \sqrt{9 - x^2}$

21. (a) Apply Newton's method to the equation $x^2 - a = 0$ to derive the following square-root algorithm (used by the ancient Babylonians to compute $\sqrt{a}$):

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

(b) Use part (a) to compute $\sqrt{1000}$ correct to six decimal places.

22. (a) Apply Newton's method to the equation $1/x - a = 0$ to derive the following reciprocal algorithm:

$$x_{n+1} = 2x_n - ax_n^2$$

(This algorithm enables a computer to find reciprocals without actually dividing.)
(b) Use part (a) to compute $1/1.6984$ correct to six decimal places.

23. Explain why Newton's method doesn't work for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial approximation is chosen to be $x_1 = 1$.

24. (a) Use Newton's method with $x_1 = 1$ to find the root of the equation $x^3 - x = 1$ correct to six decimal places.
(b) Solve the equation in part (a) using $x_1 = 0.6$ as the initial approximation.
(c) Solve the equation in part (a) using $x_1 = 0.57$. (You definitely need a programmable calculator for this part.)