**7 COROLLARY** If $f'(x) = g'(x)$ for all $x$ in an interval $(a, b)$, then $f - g$ is constant on $(a, b)$; that is, $f(x) = g(x) + c$ where $c$ is a constant.

**PROOF** Let $F(x) = f(x) - g(x)$. Then

$$F'(x) = f'(x) - g'(x) = 0$$

for all $x$ in $(a, b)$. Thus, by Theorem 5, $F$ is constant; that is, $f - g$ is constant. □

**NOTE** Care must be taken in applying Theorem 5. Let

$$f(x) = \frac{x}{|x|} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

The domain of $f$ is $D = \{x \mid x \neq 0\}$ and $f'(x) = 0$ for all $x$ in $D$. But $f$ is obviously not a constant function. This does not contradict Theorem 5 because $D$ is not an interval. Notice that $f$ is constant on the interval $(0, \infty)$ and also on the interval $(-\infty, 0)$.

We will make extensive use of Theorem 5 and Corollary 7 when we study antiderivatives in Section 4.7.

## 4.2 EXERCISES

**1–4** ▪ Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers $c$ that satisfy the conclusion of Rolle's Theorem.

**1.** $f(x) = x^2 - 4x + 1$, $[0, 4]$

**2.** $f(x) = x^3 - 3x^2 + 2x + 5$, $[0, 2]$

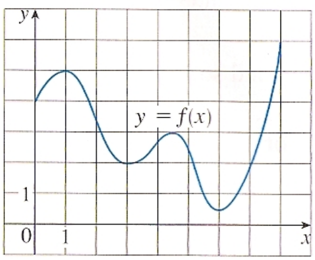
**3.** $f(x) = \sin 2\pi x$, $[-1, 1]$

**4.** $f(x) = x\sqrt{x + 6}$, $[-6, 0]$

**5.** Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number $c$ in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

**6.** Let $f(x) = (x - 1)^{-2}$. Show that $f(0) = f(2)$ but there is no number $c$ in $(0, 2)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

**7.** Use the graph of $f$ to estimate the values of $c$ that satisfy the conclusion of the Mean Value Theorem for the interval $[0, 8]$.




**8.** Use the graph of $f$ given in Exercise 7 to estimate the values of $c$ that satisfy the conclusion of the Mean Value Theorem for the interval $[1, 7]$.

**9.** (a) Graph the function $f(x) = x + 4/x$ in the viewing rectangle $[0, 10]$ by $[0, 10]$.
(b) Graph the secant line that passes through the points $(1, 5)$ and $(8, 8.5)$ on the same screen with $f$.
(c) Find the number $c$ that satisfies the conclusion of the Mean Value Theorem for this function $f$ and the interval $[1, 8]$. Then graph the tangent line at the point $(c, f(c))$ and notice that it is parallel to the secant line.

**10.** (a) In the viewing rectangle $[-3, 3]$ by $[-5, 5]$, graph the function $f(x) = x^3 - 2x$ and its secant line through the points $(-2, -4)$ and $(2, 4)$. Use the graph to estimate the $x$-coordinates of the points where the tangent line is parallel to the secant line.
(b) Find the exact values of the numbers $c$ that satisfy the conclusion of the Mean Value Theorem for the interval $[-2, 2]$ and compare with your answers to part (a).

**11–14** ▪ Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers $c$ that satisfy the conclusion of the Mean Value Theorem.

**11.** $f(x) = 3x^2 + 2x + 5$, $[-1, 1]$

**12.** $f(x) = x^3 + x - 1$, $[0, 2]$

**13.** $f(x) = e^{-2x}$, $[0, 3]$