

In Example 6 we used l'Hospital's Rule to show that

$$\lim_{x \rightarrow 0^+} x \ln x = 0$$

Therefore

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{x \ln x} = e^0 = 1$$

3.7 EXERCISES

1–36 ■ Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

1. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x + 1}$

2. $\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1}$

3. $\lim_{x \rightarrow (0/2)^+} \frac{\cos x}{1 - \sin x}$

4. $\lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x}$

5. $\lim_{t \rightarrow 0} \frac{e^t - 1}{t^3}$

6. $\lim_{t \rightarrow 0} \frac{e^{2t} - 1}{t}$

7. $\lim_{x \rightarrow 0} \frac{\tan px}{\tan qx}$

8. $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{\csc \theta}$

9. $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$

10. $\lim_{x \rightarrow \infty} \frac{\ln \ln x}{x}$

11. $\lim_{t \rightarrow 0} \frac{5^t - 3^t}{t}$

12. $\lim_{x \rightarrow \infty} \frac{\ln x}{\sin \pi x}$

13. $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$

14. $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$

15. $\lim_{x \rightarrow \infty} \frac{x}{\ln(1 + 2e^x)}$

16. $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$

17. $\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$

18. $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(4x)}$

19. $\lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2}$

20. $\lim_{x \rightarrow \infty} \frac{1 - e^{-2x}}{\sec x}$

21. $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$

22. $\lim_{x \rightarrow \infty} x^2 e^x$

23. $\lim_{x \rightarrow 0} \cot 2x \sin 6x$

24. $\lim_{x \rightarrow 0^+} \sin x \ln x$

25. $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$

26. $\lim_{x \rightarrow \infty} x \tan(1/x)$

27. $\lim_{x \rightarrow \infty} (xe^{1/x} - x)$

28. $\lim_{x \rightarrow 0} (\csc x - \cot x)$

29. $\lim_{x \rightarrow \infty} (x - \ln x)$

30. $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x - 1} \right)$

31. $\lim_{x \rightarrow 0^+} x^{x^2}$

32. $\lim_{x \rightarrow 0^+} (\tan 2x)^x$

33. $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$

34. $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} \right)^{bx}$

35. $\lim_{x \rightarrow 0^+} (\cos x)^{1/x^2}$

36. $\lim_{x \rightarrow \infty} x^{(\ln 2)/(1 + \ln x)}$

37–38 ■ Use a graph to estimate the value of the limit. Then use l'Hospital's Rule to find the exact value.

37. $\lim_{x \rightarrow \infty} x [\ln(x + 5) - \ln x]$

38. $\lim_{x \rightarrow \pi/4} (\tan x)^{\tan 2x}$

39. Prove that

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^p} = \infty$$

for any positive integer n . This shows that the exponential function approaches infinity faster than any power of x .

40. Prove that

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x .

41. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is

$$A = A_0 \left(1 + \frac{r}{n} \right)^{nt}$$

If we let $n \rightarrow \infty$, we refer to the *continuous compounding* of interest. Use l'Hospital's Rule to show that if interest is compounded continuously, then the amount after t years is

$$A = A_0 e^{rt}$$

42. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c} (1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant.

(a) Calculate $\lim_{t \rightarrow \infty} v$. What is the meaning of this limit?

(b) For fixed t , use l'Hospital's Rule to calculate $\lim_{m \rightarrow 0^+} v$.

What can you conclude about the velocity of a falling object in a vacuum?