The dispersion relation for the longitudinal oscillations of a one-dimensional chain of N identical masses m connected by springs with elastic constant C is given by

$$\omega(k) = 2\left(\frac{C}{m}\right)^{1/2}\left|\sin(ka/2)\right| ,$$

where a is the equilibrium separation of the masses.

(a) Show that the mode with wavevector $k + 2\pi/a$ has the same pattern of mass displacements as the mode with wavevector k, and hence that the dispersion relation is periodic in reciprocal space.
[Hint: When the masses are oscillating in the normal mode with wavevector k the displacement from equilibrium of the nth mass is given by $u_n(t) = A\exp\{i(kna - \omega t)\}$.]

(b) Derive expressions for the phase and group velocities, and sketch them as a function of k.

(c) Find the expression for $g(\omega)$, the density of modes per unit angular frequency. Sketch $g(\omega)$.