2. A bead of mass m slides without friction on a ring. The ring rotates with constant angular velocity ω about a rotational axis that is aligned with a ring diameter, as shown in Figure 1. Find the Lagrange equations of motion, and the Hamiltonian for the bead. Is the Hamiltonian a constant of motion? Does it coincide with the energy of the system? Interpret the Hamiltonian as the sum of the kinetic energy of a bead rotating on a fixed ring and an effective potential resulting from the gravitational potential and a centrifugal potential. Plot the effective potential as a function of the angle θ for different values of ω and discuss the motion of the bead for all cases. Under which conditions does the bead reach an equilibrium condition where θ remains constant?
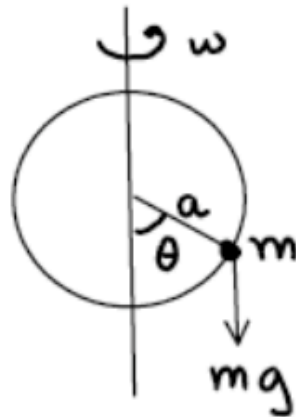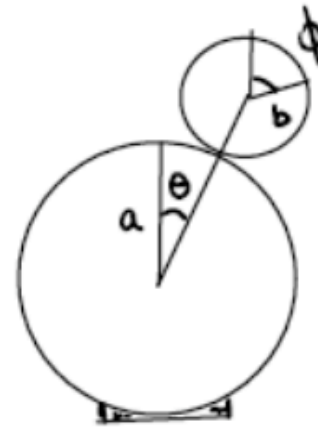

Figure 1.   Figure 2.