

2. Let $\vec{v}_1 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \\ 0 \end{pmatrix}$, and $\vec{v}_3 = \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix}$.

(a) Show that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are orthonormal.

(b) Let $\vec{w} = \begin{pmatrix} -3 \\ 9 \\ 5 \\ 9 \end{pmatrix}$. Find $\langle \vec{v}_1, \vec{w} \rangle$, $\langle \vec{v}_2, \vec{w} \rangle$, and $\langle \vec{v}_3, \vec{w} \rangle$.

(c) Use part (b) to answer the following question. Is \vec{w} a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$? If so, what is the linear combination? If not, why not?

(d) Is $\vec{u} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ -3 \end{pmatrix}$ a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$? Justify your answer.