4 Problems

21–26 ■ Find a formula for the inverse of the function.

21. $f(x) = \sqrt{10 - 3x}$

22. $f(x) = \dfrac{4x - 1}{2x + 3}$

23. $f(x) = e^{x^3}$

24. $y = 2x^3 + 3$

25. $y = \ln(x + 3)$

26. $y = \dfrac{1 + e^x}{1 - e^x}$

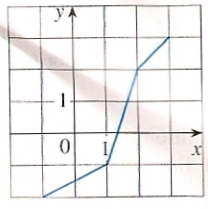
27–28 ■ Find an explicit formula for f^{-1} and use it to graph f^{-1}, f, and the line $y = x$ on the same screen. To check your work, see whether the graphs of f and f^{-1} are reflections about the line.

27. $f(x) = x^4 + 1, \quad x \geqslant 0$

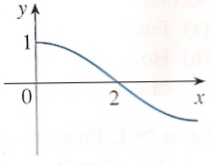
28. $f(x) = 2 - e^x$

29–30 ■ Use the given graph of f to sketch the graph of f^{-1}.

29.



30.



31–34 ■

(a) Show that f is one-to-one.
(b) Use Theorem 7 to find $(f^{-1})'(a)$.
(c) Calculate $f^{-1}(x)$ and state the domain and range of f^{-1}.
(d) Calculate $(f^{-1})'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).
(e) Sketch the graphs of f and f^{-1} on the same axes.

31. $f(x) = x^3, \quad a = 8$

32. $f(x) = \sqrt{x - 2}, \quad a = 2$

33. $f(x) = 9 - x^2, \quad 0 \leqslant x \leqslant 3, \quad a = 8$

34. $f(x) = 1/(x - 1), \quad x > 1, \quad a = 2$

35–38 ■ Find $(f^{-1})'(a)$.

35. $f(x) = x^3 + x + 1, \quad a = 1$

36. $f(x) = x^5 - x^3 + 2x, \quad a = 2$

37. $f(x) = 3 + x^2 + \tan(\pi x/2), \quad -1 < x < 1, \quad a = 3$

38. $f(x) = \sqrt{x^3 + x^2 + x + 1}, \quad a = 2$

39. Suppose f^{-1} is the inverse function of a differentiable function f and $f(4) = 5$, $f'(4) = \frac{2}{3}$. Find $(f^{-1})'(5)$.

40. Suppose f^{-1} is the inverse function of a differentiable function f and let $G(x) = 1/f^{-1}(x)$. If $f(3) = 2$ and $f'(3) = \frac{1}{9}$, find $G'(2)$.

41. (a) How is the logarithmic function $y = \log_a x$ defined?
(b) What is the domain of this function?
(c) What is the range of this function?
(d) Sketch the general shape of the graph of the function $y = \log_a x$ if $a > 1$.

42. (a) What is the natural logarithm?
(b) What is the common logarithm?
(c) Sketch the graphs of the natural logarithm function and the natural exponential function with a common set of axes.

43–46 ■ Find the exact value of each expression (without a calculator).

43. (a) $\log_2 64$ (b) $\log_6 \frac{1}{36}$

44. (a) $\log_8 2$ (b) $\ln e^{\sqrt{2}}$

45. (a) $\log_{10} 1.25 + \log_{10} 80$
(b) $\log_5 10 + \log_5 20 - 3 \log_5 2$

46. (a) $2^{(\log_2 3 + \log_2 5)}$ (b) $e^{3 \ln 2}$

47–50 ■ Use the properties of logarithms to expand the quantity.

47. $\log_2\left(\dfrac{x^3 y}{z^2}\right)$

48. $\ln \sqrt{a(b^2 + c^2)}$

49. $\ln(uv)^{10}$

50. $\ln \dfrac{3x^2}{(x + 1)^5}$

51–53 ■ Express the given quantity as a single logarithm.

51. $2 \ln 4 - \ln 2$

52. $\ln x + a \ln y - b \ln z$

53. $\ln(1 + x^2) + \frac{1}{2} \ln x - \ln \sin x$

54. Use Formula 14 to evaluate each logarithm correct to six decimal places.
(a) $\log_{12} 10$ (b) $\log_2 8.4$

55–56 ■ Use Formula 14 to graph the given functions on a common screen. How are these graphs related?

55. $y = \log_{1.5} x, \quad y = \ln x, \quad y = \log_{10} x, \quad y = \log_{50} x$

56. $y = \ln x, \quad y = \log_{10} x, \quad y = e^x, \quad y = 10^x$

57. Suppose that the graph of $y = \log_2 x$ is drawn on a coordinate grid where the unit of measurement is an inch. How many miles to the right of the origin do we have to move before the height of the curve reaches 3 ft?

58. Compare the functions $f(x) = x^{0.1}$ and $g(x) = \ln x$ by graphing both f and g in several viewing rectangles. When does the graph of f finally surpass the graph of g?