Therefore, the relative error in the volume is approximately three times the relative error in the radius. In Example 3 the relative error in the radius is approximately $dr/r = 0.05/21 \approx 0.0024$ and it produces a relative error of about 0.007 in the volume. The errors could also be expressed as **percentage errors** of 0.24% in the radius and 0.7% in the volume.

3 problems

2.8 EXERCISES

1–4 ▪ Find the linearization $L(x)$ of the function at a.

1. $f(x) = x^4 + 3x^2, \quad a = -1$

2. $f(x) = 1/\sqrt{2 + x}, \quad a = 0$

3. $f(x) = \cos x, \quad a = \pi/2$

4. $f(x) = x^{3/4}, \quad a = 16$

5. Find the linear approximation of the function $f(x) = \sqrt{1 - x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by graphing f and the tangent line.

6. Find the linear approximation of the function $g(x) = \sqrt[3]{1 + x}$ at $a = 0$ and use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$. Illustrate by graphing g and the tangent line.

7–10 ▪ Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1.

7. $\sqrt[3]{1 - x} \approx 1 - \frac{1}{3}x$

8. $\tan x \approx x$

9. $1/(1 + 2x)^4 \approx 1 - 8x$

10. $1/\sqrt{4 - x} \approx \frac{1}{2} + \frac{1}{16}x$

11–14 ▪ Use a linear approximation (or differentials) to estimate the given number.

11. $(2.001)^5$

12. $\sqrt{99.8}$

13. $(8.06)^{2/3}$

14. $1/1002$

15–16 ▪ Explain, in terms of linear approximations or differentials, why the approximation is reasonable.

15. $\sec 0.08 \approx 1$

16. $(1.01)^6 \approx 1.06$

17–18 ▪ Find the differential of each function.

17. (a) $y = x^2 \sin 2x$ (b) $y = \sqrt{4 + 5x}$

18. (a) $y = s/(1 + 2s)$ (b) $y = 1/(x + 1)$

19. Let $y = \tan x$.
(a) Find the differential dy.
(b) Evaluate dy and Δy if $x = \pi/4$ and $dx = -0.1$.

20. Let $y = \sqrt{x}$.
(a) Find the differential dy.
(b) Evaluate dy and Δy if $x = 1$ and $dx = \Delta x = 1$.
(c) Sketch a diagram like Figure 5 showing the line segments with lengths dx, dy, and Δy.

21. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing (a) the volume of the cube and (b) the surface area of the cube.

22. The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm.
(a) Use differentials to estimate the maximum error in the calculated area of the disk.
(b) What is the relative error? What is the percentage error?

23. The circumference of a sphere was measured to be 84 cm with a possible error of 0.5 cm.
(a) Use differentials to estimate the maximum error in the calculated surface area. What is the relative error?
(b) Use differentials to estimate the maximum error in the calculated volume. What is the relative error?

24. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with diameter 50 m.

25. When blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:

$$F = kR^4$$

(This is known as Poiseuille's Law.) A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow.

Show that the relative change in F is about four times the relative change in R. How will a 5% increase in the radius affect the flow of blood?

26. On page 431 of *Physics: Calculus,* 2d ed., by Eugene Hecht (Pacific Grove, CA: Brooks/Cole, 2000), in the course of