An economist wishes to estimate the average family income in a certain population. The population standard deviation is known to be $4,500, and the economist uses a random sample of size = 225.

**Solution:**

Since the sample size is very large, and by the central limit theorem, the sample mean, is approximately normal random variable, with mean equal to the population mean, and standard deviation.

To find the probability in (a) and (b), we need to transform  to the standard normal random variable, Z, which has mean of 0 and standard deviation of 1, such that we can check the z-table.

<http://lilt.ilstu.edu/dasacke/eco138/ZTable.htm>

1. What is the probability that the sample mean will fall within $800 of the population mean?

This is the probability that the distance between the sample mean and the population mean is less than or equal to $800.



By checking z-table

=0.9962 - 0.0038

=0.9924

Can you please explain how you get from P(Z<8/3) – P(Z<-8/3) to 0.9962 – 0.0038? How do you check the z-table to find these values? Please show me in the Example Z-Table below.

1. What is the probability that the sample mean will exceed the population mean by more than $600.



Can you please explain to me how you get from 1-P(Z<2) to 1-0.9772 to 0.0278 in detail? Should 0.0278 really equal 0.0228? Please show me in the Example Z-Table below.

The article “Reliability of Domestic Waste Biofilm Reactors” (*J. of Envir. Engr.,* 1995: 785-790) suggests that substrate concentration (mg/cm^3) of influent to a reactor is normally distributed with μ = 0.30 and σ = 0.06.

**Solution:**

Let X be the concentration of influent. As in the previous problem, we need to transform X to the standard normal random variable, Z, by subtracting its mean and divided by standard deviation.

* 1. What is the probability that the concentration exceeds 0.40?



Same question as above, how do you find 0.9525 from the z-tables? Please show me in the Example Z-Table below.

* 1. What is the probability that the concentration is at most 0.25?



Same question as above, how do you find P(Z<= -5/6) from the z-tables? Please show me in the Example Z-Table below.

* 1. How would you characterize the largest 10% of all concentration levels?

We denote this by x0, which satisfies that

P(X<x0)=0.90, that is, 90% of the concentration level is below this point. In other words, x0 is the cutoff value of top 10% concentration level.

To find x0, we still need to transform to standard format



If you look at the z-table, look for probability 0.9 among the internal cells, and the closest probability is 0.8997, corresponding to the z-value of 1.28.

Please show me in the z-table below where you found these values? Please show me in the Example Z-Table below.

Therefore

. This tells you that

 or



or

x0= 0.3+1.28\*0.06=0.3768

Therefore the largest 10% of all concentration levels is 0.3768.

Remark: Since normal distribution is continuous, which means , or more specifically, P(X=x)=0 (the probability of X falling on a single point is zero). We do not care if the inequality is strict or not in all our probabilities.

For the discontinuous distribution, such at binomial, P(X=x), however, is not zero.

Example Z-Table

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0 | 0 | 0.004 | 0.008 | 0.012 | 0.016 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.091 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.148 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.17 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.195 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.219 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.258 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.291 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.334 | 0.3365 | 0.3389 |
| 1 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.377 | 0.379 | 0.381 | 0.383 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.398 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.437 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.475 | 0.4756 | 0.4761 | 0.4767 |
| 2 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.483 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.485 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.489 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.492 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.494 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.496 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.497 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.498 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.499 | 0.499 |
| 3.1 | 0.499 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |