

Eight-Hour Work Schedule

1	2	3	4	5	6	7	8
Work	Work	Break	Work	Work	Break	Work	Work

The company would like to determine how many drivers to call in at each hour of the day. Drivers begin their shifts on the hour (at 6, 7, 8, . . . , 16). No drivers are called after hour 16. There must be at least enough drivers scheduled to cover the hourly requirements under the condition that each follows the 8-hour pattern given in the table. The goal is to minimize the total number of drivers used. Formulate and solve an LP model for this problem.

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Manufacturing Data						
Period	Plant A			Plant B		
	Unit cost (\$/unit)	Capacity		Unit cost (\$/unit)	Capacity	
1	8	175		7	200	
2	10	150		8	170	

Demand Data						
Period	Selling price (\$/unit)			Maximum sales		
	I	II	III	I	II	III
1	15	20	14	100	200	150
2	18	17	21	150	300	150

14. A company is planning its aggregate production schedule for the next 3 months. Units may be produced on regular time or overtime. The relevant costs and capacities are shown in the table. The demand for each month is also shown. There are three ways of meeting this demand: inventory, current production, and back-orders. Units produced in a particular month may be sold in that month or kept in inventory for sale in a later month. There is a \$1 cost per unit for each month an item is held in inventory. Initially, there are 15 units in inventory. Also, sales can be back-ordered at a cost of \$2 per unit per month. Back-orders represent production in future months to satisfy demand in past months, and hence incur an additional cost. Initially there are no back-orders. There should be no inventory or back-orders after month 3.

Month	Capacity (units)		Production cost (\$/unit)		
	Regular time	Overtime	Regular time	Overtime	Demand
1	100	20	14	18	60
2	100	10	17	22	80
3	60	20	17	22	140

- (a) Develop a model that when solved will yield the optimal production plan. Use only one variable for the inventory and one variable for back-orders for each month. Find the solution with a computer program.
- (b) How would the model change if the inventory cost depended on the total time an item was stored? Let the cost be \$1 per unit for items kept in inventory for 1 month, \$3 for items kept for 2 months, and \$5 for items kept for 3 months. Assume that the initial inventory has been in storage for 1 month. You will need more than one inventory variable for each month. Solve the model given these conditions.
15. A shuttle bus system operates at a university from 6 A.M. to midnight, a period of 18 hours. To meet student demands for service, the following schedule has been determined for the number of bus drivers required during each hour of the day. Times are stated with respect to a 24-hour clock.

Time Period:	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15
Number of drivers:	5	20	25	20	15	12	25	20	15
Time Period:	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Number of drivers:	15	12	20	20	18	15	10	5	3

During an 8-hour day, each driver receives two 1-hour breaks. The 8-hour period is divided between work and breaks, as shown in the following table.