1. Show that the commutator obeys:
   a. $[A, B] = -[B, A]$
   b. $[A, B + C] = [A, B] + [A, C]$
   c. $[A, BC] = [A, B]C + B[A, C]$
   d. $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$

2. Given the fundamental commutator relation between momentum and position: $[\hat{x}, \hat{p}] = \imath\hbar$, show that i) $[\hat{x}^n, \hat{p}] = \imath\hbar n\hat{x}^{n-1}$, ii) $[\hat{x}, \hat{p}^n] = \imath\hbar n\hat{p}^{n-1}$. Using these results show that if $f(\hat{x})$ can be expanded in a polynomial in $\hat{x}$ and $g(\hat{p})$ can be expanded in a polynomial in $\hat{p}$, then $[f(\hat{x}), \hat{p}] = \imath\hbar df/d\hat{x}$, and $[x, g(\hat{p})] = \imath\hbar dg/d\hat{p}$.

3. The Hamiltonian operator $\hat{H}$ for a certain physical system is represented by the matrix

$$\hat{H} = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{1}$$

while two other observables A and B are represented by the matrices

$$\hat{A} = \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 2\lambda \end{pmatrix}, \qquad \hat{B} = \begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 0 & \mu \\ 0 & \mu & 0 \end{pmatrix}, \tag{2}$$

where λ and μ are real numbers.

a. Find the eigenvalues and eigenvectors of A and B.

b. If the system is in a state described by the state vector

$$\mathbf{u} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3 \tag{3}$$

where c_1, c_2, and c_3 are constants and

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{4}$$

i. Find the relationship between c_1, c_2, and c_3 such that $\mathbf{u}$ is normalized to unity.

ii. Find the expectation value of H, A, and B.

iii. What are the possible values of the energy that can be obtained in a measurement when the system is described by the vector $\mathbf{u}$? and with what probability?