

Exercise 1.6.4. * It is assumed that you know (1) what a *determinant* is, (2) that $\det \Omega^T = \det \Omega$ (T denotes transpose), (3) that the determinant of a product of matrices is the product of the determinants. [If you do not, verify these properties for a two-dimensional case

$$\Omega = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

with $\det \Omega = (a\delta - \beta\gamma)$.] Prove that the determinant of a unitary matrix is a complex number of unit modulus.

Exercise 1.6.5. * Verify that $R(\frac{1}{2}\pi i)$ is unitary (orthogonal) by examining its matrix.

Exercise 1.6.6. Verify that the following matrices are unitary:

$$\frac{1}{2^{1/2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}, \quad \frac{1}{2} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$

Verify that the determinant is of the form $e^{i\theta}$ in each case. Are any of the above matrices Hermitian?