

Exercise 1.8.3.* Consider the Hermitian matrix

$$\Omega = \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

- (1) Show that $\omega_1 = \omega_2 = 1$; $\omega_3 = 2$.
- (2) Show that $|\omega = 2\rangle$ is any vector of the form

$$\frac{1}{(2a^2)^{1/2}} \begin{bmatrix} 0 \\ a \\ -a \end{bmatrix}$$

- (3) Show that the $\omega = 1$ eigenspace contains all vectors of the form

$$\frac{1}{(b^2 + 2c^2)^{1/2}} \begin{bmatrix} b \\ c \\ c \end{bmatrix}$$

either by feeding $\omega = 1$ into the equations or by requiring that the $\omega = 1$ eigenspace be orthogonal to $|\omega = 2\rangle$.