*Exercise 1.8.5.** Consider the matrix

$$\Omega = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

(1) Show that it is unitary.
(2) Show that its eigenvalues are $e^{i\theta}$ and $e^{-i\theta}$.
(3) Find the corresponding eigenvectors; show that they are orthogonal.
(4) Verify that $U^\dagger \Omega U$ = (diagonal matrix), where U is the matrix of eigenvectors of Ω.

*Exercise 1.8.6.** (1) We have seen that the determinant of a matrix is unchanged under a unitary change of basis. Argue now that

$$\det \Omega = \text{product of eigenvalues of } \Omega = \prod_{i=1}^{n} \omega_i$$

for a Hermitian or unitary Ω.

(2) Using the invariance of the trace under the same transformation, show that

$$\operatorname{Tr} \Omega = \sum_{i=1}^{n} \omega_i$$

Exercise 1.8.7. By using the results on the trace and determinant from the last problem, show that the eigenvalues of the matrix

$$\Omega = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

are 3 and −1. Verify this by explicit computation. Note that the Hermitian nature of the matrix is an essential ingredient.