The linear programming problem can now be stated as follows:

$$\begin{aligned} &\text{Maximize} && z = 120x_1 + 40x_2 + 60x_3 \\ &\text{subject to:} && x_1 + x_2 + x_3 + s_1 = 100 \\ & && 10x_1 + 4x_2 + 7x_3 + s_2 = 500 \\ &\text{with} && x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad s_1 \geq 0, \quad s_2 \geq 0. \end{aligned}$$

Rewrite the objective function as $-120x_1 - 40x_2 - 60x_3 + z = 0$, and complete the initial simplex tableau as follows.

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & s_1 & s_2 & z & \end{matrix} \\ \left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 100 \\ 10 & 4 & 7 & 0 & 1 & 0 & 500 \\ \hline -120 & -40 & -60 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

The maximization problem in Example 2 consists of a system of two equations (describing the constraints) in five variables, together with the objective function. As with the graphical method, it is necessary to solve this system to find corner points of the region of feasible solutions. To produce a single, distinct solution, the number of variables must equal the number of independent equations in the system.

For example, the system of equations in Example 2 has an infinite number of solutions since there are more variables than equations. To see this, solve the system for s_1 and s_2.

$$\begin{aligned} s_1 &= 100 - x_1 - x_2 - x_3 \\ s_2 &= 500 - 10x_1 - 4x_2 - 7x_3 \end{aligned}$$

Each choice of values for x_1, x_2, and x_3 gives corresponding values for s_1 and s_2 that produce a solution of the system. But only some of these solutions are feasible. In a feasible solution all variables must be nonnegative. To get a unique feasible solution, we set three of the five variables equal to 0. In general, if there are m equations, then m variables can be nonzero. These m nonzero variables are called **basic variables,** and the corresponding solutions are called **basic feasible solutions.** Each basic feasible solution corresponds to a corner point. In particular, if we choose the solution with $x_1 = 0$, $x_2 = 0$, and $x_3 = 0$, then $s_1 = 100$ and $s_2 = 500$ are the basic variables. This solution, which corresponds to the corner point at the origin, is hardly optimal. It produces a profit of \$0 for the farmer, since the equation that corresponds to the objective function becomes

$$-120(0) - 40(0) - 60(0) + 0s_1 + 0s_2 + z = 0.$$

In the next section we will use the simplex method to start with this solution and improve it to find the maximum possible profit.

Each step of the simplex method produces a solution that corresponds to a corner point of the region of feasible solutions. These solutions can be read directly from the matrix, as shown in the next example.