

This matrix is called the initial **simplex tableau**. The numbers in the bottom row, which are from the objective function, are called **indicators** (except for the 1 and 0 at the far right).

### EXAMPLE 2 Initial Simplex Tableau

Set up the initial simplex tableau for the following problem.

A farmer has 100 acres of available land that he wishes to plant with a mixture of potatoes, corn, and cabbage. It costs him \$400 to produce an acre of potatoes, \$160 to produce an acre of corn, and \$280 to produce an acre of cabbage. He has a maximum of \$20,000 to spend. He makes a profit of \$120 per acre of potatoes, \$40 per acre of corn, and \$60 per acre of cabbage. How many acres of each crop should he plant to maximize his profit?

**Solution** Begin by summarizing the given information as follows.

	Potatoes	Corn	Cabbage	Total
Number of Acres	$x_1$	$x_2$	$x_3$	$\leq 100$
Cost (per acre)	\$400	\$160	\$280	$\leq \$20,000$
Profit (per acre)	\$120	\$40	\$60	

If the number of acres allotted to each of the three crops is represented by  $x_1$ ,  $x_2$ , and  $x_3$ , respectively, then the constraints of the example can be expressed as

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 100 && \text{Number of acres} \\ 400x_1 + 160x_2 + 280x_3 &\leq 20,000 && \text{Production costs} \end{aligned}$$

where  $x_1$ ,  $x_2$ , and  $x_3$  are all nonnegative. The first of these constraints says that  $x_1 + x_2 + x_3$  is less than or perhaps equal to 100. Use  $s_1$  as the slack variable, giving the equation

$$x_1 + x_2 + x_3 + s_1 = 100.$$

Here  $s_1$  represents the amount of the farmer's 100 acres that will not be used ( $s_1$  may be 0 or any value up to 100).

The constraint  $400x_1 + 160x_2 + 280x_3 \leq 20,000$  can be simplified by dividing by 40, to get

$$10x_1 + 4x_2 + 7x_3 \leq 500.$$

This inequality can also be converted into an equation by adding a slack variable,  $s_2$ .

$$10x_1 + 4x_2 + 7x_3 + s_2 = 500$$

If we had not divided by 40, the slack variable would have represented any unused portion of the farmer's \$20,000 capital. Instead, the slack variable represents  $1/40$  of that unused portion. (Note that  $s_2$  may be any value from 0 to 500.)

The objective function represents the profit. The farmer wants to maximize

$$z = 120x_1 + 40x_2 + 60x_3.$$