6. Show that the Legendre polynomials satisfy

$$\int_0^1 P_k(x)\,\mathrm{d}x = \begin{cases} 1 & k = 0 \\ \frac{(-1)^{(k-1)/2}\,(k-1)!}{2^k\,(\frac{k+1}{2})!\,(\frac{k-1}{2})!} & k \text{ odd} \\ 0 & \text{otherwise.} \end{cases}$$

Use your result to expand $|x|$ in the domain $-1 < x < 1$ in Legendre polynomials.