4. Prove the convolution theorem

$$\int_{-\infty}^{\infty} f(x)\, g(y-x)\, \mathrm{d}x = \int_{-\infty}^{\infty} \tilde{f}(k)\, \tilde{g}(k)\, e^{iky}\, \mathrm{d}k,$$

for the Fourier transform. Deduce from it Parseval's relation that the L^2-norm of any function $f(x)$ is equal to that of its Fourier transform $\tilde{f}(k)$.