

Exercise 2.8.7. Consider the harmonic oscillator, for which the general solution is

$$x(t) = A \cos \omega t + B \sin \omega t.$$

Express the energy in terms of A and B and note that it does not depend on time. Now choose A and B such that $x(0) = x_1$ and $x(T) = x_2$. Write down the energy in terms of x_1 , x_2 , and T . Show that the action for the trajectory connecting x_1 and x_2 is

$$S_{cl}(x_1, x_2, T) = \frac{m\omega}{2 \sin \omega T} [(x_1^2 + x_2^2) \cos \omega T - 2x_1 x_2].$$

Verify that $\partial S_{cl} / \partial T = -E$.