5. **Train Travel.** A train leaves Barstow traveling east at 35 km/h. One hour later, a faster train leaves Barstow, also traveling east on a parallel track at 40 km/h. How far from Barstow will the faster train catch up with the slower one?

$d = r \cdot t$

	DISTANCE	RATE	TIME	
Slow Train			t	$\rightarrow d =$
Fast Train	d			$\rightarrow d =$

We organize the information in a table as follows.

$d = r \cdot t$

	DISTANCE	RATE	TIME	
Brother	d	55	$t + 1$	$\rightarrow d = 55(t + 1)$
You	d	65	t	$\rightarrow d = 65t$

2. **Translate.** Using $d = rt$ in each row of the table, we get an equation. Thus we have a system of equations:

$$d = 55(t + 1), \quad (1)$$
$$d = 65t. \quad (2)$$

3. **Solve.** We solve the system using the substitution method:

$65t = 55(t + 1)$ Substituting $65t$ for d in equation (1)

$65t = 55t + 55$ Multiplying to remove parentheses on the right

$\left.\begin{aligned} 10t &= 55 \\ t &= 5.5. \end{aligned}\right\}$ Solving for t

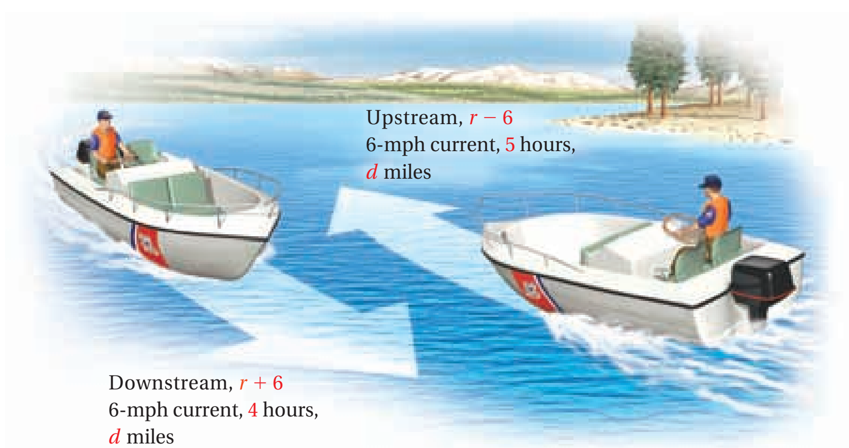
Your time is 5.5 hr, which means that your brother's time is 5.5 + 1, or 6.5 hr.

4. **Check.** At 65 mph, you will travel $65 \cdot 5.5$, or 357.5 mi, in 5.5 hr. At 55 mph, your brother will travel $55 \cdot 6.5$, or the same 357.5 mi, in 6.5 hr. The numbers check.

5. **State.** You will catch up with your brother in 5.5 hr.

Do Exercise 5.

EXAMPLE 6 *Marine Travel.* A Coast-Guard patrol boat travels 4 hr on a trip downstream with a 6-mph current. The return trip against the same current takes 5 hr. Find the speed of the boat in still water.



Answer on page A-36

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1. **Familiarize.** We first make a drawing. From the drawing, we see that the distances are the same. We let $d =$ the distance, in miles, and $r =$ the speed of the boat in still water, in miles per hour. Then, when the boat is traveling downstream, its speed is $r + 6$ (the current helps the boat along). When it is traveling upstream, its speed is $r - 6$ (the current holds the boat back). We can organize the information in a table. We use the formula $d = rt$.

	d =	r ·	t	
	DISTANCE	**RATE**	**TIME**	
Downstream	d	$r + 6$	4	$\rightarrow d = (r + 6)4$
Upstream	d	$r - 6$	5	$\rightarrow d = (r - 6)5$

2. **Translate.** From each row of the table, we get an equation, $d = rt$:

$$d = 4r + 24, \qquad (1)$$
$$d = 5r - 30. \qquad (2)$$

3. **Solve.** We solve the system by the substitution method:

$4r + 24 = 5r - 30$ Substituting $4r + 24$ for d in equation (2)

$\left.\begin{aligned} 24 &= r - 30 \\ 54 &= r. \end{aligned}\right\}$ Solving for r

4. **Check.** If $r = 54$, then $r + 6 = 60$; and $60 \cdot 4 = 240$, the distance traveled downstream. If $r = 54$, then $r - 6 = 48$; and $48 \cdot 5 = 240$, the distance traveled upstream. The distances are the same. In this type of problem, a problem-solving tip to keep in mind is "Have I found what the problem asked for?" We could solve for a certain variable but still have not answered the question of the original problem. For example, we might have found speed when the problem wanted distance. In this problem, we want the speed of the boat in still water, and that is r.

5. **State.** The speed in still water is 54 mph.

Do Exercise 6.

6. **Air Travel.** An airplane flew for 4 hr with a 20-mph tailwind. The return flight against the same wind took 5 hr. Find the speed of the plane in still air.

	d =	r ·	t	
	DISTANCE	**RATE**	**TIME**	
With Wind		$r + 20$		$\rightarrow d =$
Against Wind	d			$\rightarrow d =$

Answer on page A-36

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Translating for Success

1. *Office Expense.* The monthly phone expense for an office is \$1094 less than the janitorial expense. Three times the janitorial expense minus four times the phone expense is \$248. What is the total of the two expenses?

2. *Dimensions of a Triangle.* The sum of the base and the height of a triangle is 192 in. The height is twice the base. Find the base and the height.

3. *Supplementary Angles.* Two supplementary angles are such that twice one angle is 7° more than the other. Find the measures of the angles.

4. *SAT Scores.* The total of Megan's verbal and math scores on the SAT was 1094. Her math score was 248 points higher than her verbal score. What were her math and verbal SAT scores?

5. *Sightseeing Boat.* A sightseeing boat travels 3 hr on a trip downstream with a 2.5-mph current. The return trip against the same current takes 3.5 hr. Find the speed of the boat in still water.

The goal of these matching questions is to practice step (2), *Translate,* of the five-step problem-solving process. Translate each word problem to a system of equations and select a correct translation from systems A–J.

A. $x = y + 248,$
$x + y = 1094$

B. $5x = 2y - 3,$
$y = \frac{2}{3}x + 5$

C. $y = \frac{1}{2}x,$
$2x + 2y = 192$

D. $2x = 7 + y,$
$x + y = 180$

E. $x + y = 192,$
$x = 2y$

F. $x + y = 180,$
$x = 2y + 7$

G. $x - 1094 = y,$
$3x - 4y = 248$

H. $3\%x + 2.5\%y = 97.50,$
$x + y = 2500$

I. $2x = 5 + \frac{2}{3}y,$
$3y = 15x - 4$

J. $x(y + 2.5) \cdot 3,$
$3.5(y - 2.5) = x$

Answers on page A-36

6. *Running Distances.* Each day Tricia runs 5 miles more than two-thirds the distance that Chris runs. Five times the distance that Chris runs is 3 mi less than twice the distance that Tricia runs. How far does Tricia run daily?

7. *Dimensions of a Rectangle.* The perimeter of a rectangle is 192 in. The width is half the length. Find the length and the width.

8. *Mystery Numbers.* Teka asked her students to determine the two numbers that she placed in a sealed envelope. Twice the smaller number is 5 more than two-thirds of the larger number. Three times the larger number is 4 less than fifteen times the smaller. Find the numbers.

9. *Supplementary Angles.* Two supplementary angles are such that one angle is 7° more than twice the other. Find the measures of the angles.

10. *Student Loans.* Brandt's student loans totaled \$2500. Part was made at 3% interest and the rest at 2.5%. After one year, Brandt had accumulated \$97.50 in interest. What was the amount of each loan?

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8.4 EXERCISE SET

For Extra Help

MathXL MyMathLab InterAct Math Math Tutor Center Digital Video Tutor CD 4 Videotape 8 Student's Solutions Manual

a Solve.

1. *Retail Sales.* Paint Town sold 45 paintbrushes, one kind at \$8.50 each and another at \$9.75 each. In all, \$398.75 was taken in for the brushes. How many of each kind were sold?

2. *Retail Sales.* Mountainside Fleece sold 40 neckwarmers. Solid-color neckwarmers sold for \$9.90 each and print ones sold for \$12.75 each. In all, \$421.65 was taken in for the neckwarmers. How many of each type were sold?

3. *Sales of Pharmaceuticals.* In 2004, the Diabetic Express charged \$27.06 for a vial of Humulin insulin and \$34.39 for a vial of Novolin Velosulin insulin. If a total of \$1565.57 was collected for 50 vials of insulin, how many vials of each type were sold?

4. *Fundraising.* The St. Mark's Community Barbecue served 250 dinners. A child's plate cost \$3.50 and an adult's plate cost \$7.00. A total of \$1347.50 was collected. How many of each type of plate was served?

5. *Radio Airplay.* Rudy must play 12 commercials during his 1-hr radio show. Each commercial is either 30 sec or 60 sec long. If the total commercial time during that hour is 10 min, how many commercials of each type does Rudy play?

6. *Nontoxic Floor Wax.* A nontoxic floor wax can be made by combining lemon juice and food-grade linseed oil. The amount of oil should be twice the amount of lemon juice. How much of each ingredient is needed in order to make 32 oz of floor wax? (The mix should be spread with a rag and buffed when dry.)

7. *Catering.* Stella's Catering is planning a wedding reception. The bride and groom would like to serve a nut mixture containing 25% peanuts. Stella has available mixtures that are either 40% or 10% peanuts. How much of each type should be mixed to get a 10-lb mixture that is 25% peanuts?

8. *Blending Granola.* Deep Thought Granola is 25% nuts and dried fruit. Oat Dream Granola is 10% nuts and dried fruit. How much of Deep Thought and how much of Oat Dream should be mixed to form a 20-lb batch of granola that is 19% nuts and dried fruit?

9. *Ink Remover.* Etch Clean Graphics uses one cleanser that is 25% acid and a second that is 50% acid. How many liters of each should be mixed to get 10 L of a solution that is 40% acid?

10. *Livestock Feed.* Soybean meal is 16% protein and corn meal is 9% protein. How many pounds of each should be mixed to get a 350-lb mixture that is 12% protein?

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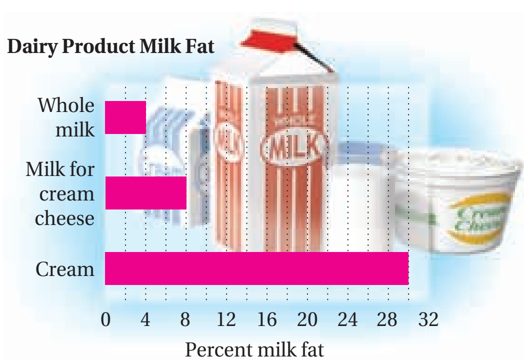
11. *Dry Cleaners.* Claudio, a banking vice-president, took 17 neckties to Milto Cleaners. The rate for non-silk ties is \$3.25 per tie and for silk ties is \$3.60 per tie. His total bill was \$58.75. How many silk ties did he have dry cleaned?

12. *Laundry.* While on a four-week hiking trip in the mountains, the Tryon family washed 11 loads of clothes at The Mountain View Laundry. The 20-lb capacity washing machine costs \$1.50 per load while the 30-lb costs \$2.50. Their total laundry expense was \$20.50. How many loads were laundered in each size washing machine?

13. *Student Loans.* Sarah's two student loans totaled \$12,000. One of her loans was at 6% simple interest and the other at 9%. After one year, Sarah owed \$855 in interest. What was the amount of each loan?

14. *Investments.* An executive nearing retirement made two investments totaling \$45,000. In one year, these investments yielded \$2430 in simple interest. Part of the money was invested at 4% and the rest at 6%. How much was invested at each rate?

15. *Food Science.* The following bar graph shows the milk fat percentages in three dairy products. How many pounds each of whole milk and cream should be mixed to form 200 lb of milk for cream cheese?



16. *Automotive Maintenance.* "Arctic Antifreeze" is 18% alcohol and "Frost No-More" is 10% alcohol. How many liters of Arctic Antifreeze should be mixed with 7.5 L of Frost No-More in order to get a mixture that is 15% alcohol?

17. *Teller Work.* Juan goes to a bank and gets change for a \$50 bill consisting of all \$5 bills and \$1 bills. There are 22 bills in all. How many of each kind are there?

18. *Making Change.* Christina makes a \$9.25 purchase at a bookstore in Reno with a \$20 bill. The store has no bills and gives her the change in quarters and fifty-cent pieces. There are 30 coins in all. How many of each kind are there?

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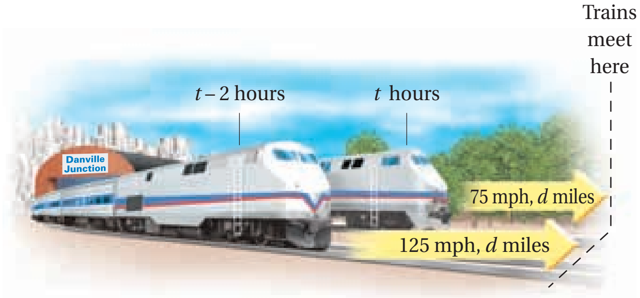
19. *Investments.* William opened two investment accounts for his grandson's college fund. The first year, these investments, which totaled \$18,000, yielded \$831 in simple interest. Part of the money was invested at 5.5% and the rest at 4%. How much was invested at each rate?

20. *Student Loans.* Cole's two student loans totaled \$31,000. One of his loans was at 2.8% simple interest and the other at 4.5%. After one year, Cole owed \$1024.40 in interest. What was the amount of each loan?

b Solve.

21. *Train Travel.* A train leaves Danville Junction and travels north at a speed of 75 mph. Two hours later, a second train leaves on a parallel track and travels north at 125 mph. How far from the station will they meet?



22. *Car Travel.* Two cars leave Denver traveling in opposite directions. One car travels at a speed of 80 km/h and the other at 96 km/h. In how many hours will they be 528 km apart?

23. *Canoeing.* Darren paddled for 4 hr with a 6-km/h current to reach a campsite. The return trip against the same current took 10 hr. Find the speed of Darren's canoe in still water.

24. *Boating.* Mia's motorboat took 3 hr to make a trip downstream with a 6-mph current. The return trip against the same current took 5 hr. Find the speed of the boat in still water.

25. *Car Travel.* Donna is late for a sales meeting after traveling from one town to another at a speed of 32 mph. If she had traveled 4 mph faster, she could have made the trip in $\frac{1}{2}$ hr less time. How far apart are the towns?

26. *Air Travel.* Rod is a pilot for Crossland Airways. He computes his flight time against a headwind for a trip of 2900 mi at 5 hr. The flight would take 4 hr and 50 min if the headwind were half as great. Find the headwind and the plane's air speed.

27. *Air Travel.* Two planes travel toward each other from cities that are 780 km apart at rates of 190 km/h and 200 km/h. They started at the same time. In how many hours will they meet?

28. *Motorcycle Travel.* Sally and Rocky travel on motorcycles toward each other from Chicago and Indianapolis, which are about 350 km apart, and they are biking at rates of 110 km/h and 90 km/h. They started at the same time. In how many hours will they meet?

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29. *Air Travel.* Two airplanes start at the same time and fly toward each other from points 1000 km apart at rates of 420 km/h and 330 km/h. After how many hours will they meet?

30. *Truck and Car Travel.* A truck and a car leave a service station at the same time and travel in the same direction. The truck travels at 55 mph and the car at 40 mph. They can maintain CB radio contact within a range of 10 mi. When will they lose contact?

31. *Point of No Return.* A plane flying the 3458-mi trip from New York City to London has a 50-mph tailwind. The flight's *point of no return* is the point at which the flight time required to return to New York is the same as the time required to continue to London. If the speed of the plane in still air is 360 mph, how far is New York from the point of no return?

32. *Point of No Return.* A plane is flying the 2553-mi trip from Los Angeles to Honolulu into a 60-mph headwind. If the speed of the plane in still air is 310 mph, how far from Los Angeles is the plane's point of no return? (See Exercise 31.)

33. D_W List three or four study tips for someone beginning this exercise set.

34. D_W Write a problem similar to Margin Exercise 1 for a classmate to solve. Design the problem so the answer is "The florist sold 14 hanging plants and 9 flats of petunias."

SKILL MAINTENANCE

Given the function $f(x) = 4x - 7$, find each of the following function values. [7.1b]

35. $f(0)$

36. $f(-1)$

37. $f(1)$

38. $f(10)$

39. $f(-2)$

40. $f(2a)$

41. $f(-4)$

42. $f(1.8)$

43. $f\left(\frac{3}{4}\right)$

44. $f(-2.5)$

45. $f(-3h)$

46. $f(1000)$

SYNTHESIS

47. *Automotive Maintenance.* The radiator in Michelle's car contains 16 L of antifreeze and water. This mixture is 30% antifreeze. How much of this mixture should she drain and replace with pure antifreeze so that there will be a mixture of 50% antifreeze?

48. *Physical Exercise.* Natalie jogs and walks to school each day. She averages 4 km/h walking and 8 km/h jogging. The distance from home to school is 6 km and Natalie makes the trip in 1 hr. How far does she jog in a trip?

49. *Fuel Economy.* Sally Cline's SUV gets 18 miles per gallon (mpg) in city driving and 24 mpg in highway driving. The SUV is driven 465 mi on 23 gal of gasoline. How many miles were driven in the city and how many were driven on the highway?

50. *Gender.* Phil and Phyllis are siblings. Phyllis has twice as many brothers as she has sisters. Phil has the same number of brothers as sisters. How many girls and how many boys are in the family?

51. *Wood Stains.* Bennet Custom Flooring has 0.5 gal of stain that is 20% brown and 80% neutral. A customer orders 1.5 gal of a stain that is 60% brown and 40% neutral. How much pure brown stain and how much neutral stain should be added to the original 0.5 gal in order to make up the order?

52. See Exercise 51. Let $x =$ the amount of pure brown stain added to the original 0.5 gal. Find a function $P(x)$ that can be used to determine the percentage of brown stain in the 1.5-gal mixture. On a graphing calculator, draw the graph of P and use ZOOM and TRACE or the TABLE feature to confirm the answer to Exercise 51.

8.5 SYSTEMS OF EQUATIONS IN THREE VARIABLES

Objective

a Solve systems of three equations in three variables.

a Solving Systems in Three Variables

A **linear equation in three variables** is an equation equivalent to one of the type $Ax + By + Cz = D$. A **solution** of a system of three equations in three variables is an ordered triple (x, y, z) that makes *all three* equations true.

The substitution method can be used to solve systems of three equations, but it is not efficient unless a variable has already been eliminated from one or more of the equations. Therefore, we will use only the elimination method—essentially the same procedure for systems of three equations as for systems of two equations.* The first step is to eliminate a variable and obtain a system of two equations in two variables.

EXAMPLE 1 Solve the following system of equations:

$$\begin{aligned} x + y + z &= 4, & \quad (1)\\ x - 2y - z &= 1, & \quad (2)\\ 2x - y - 2z &= -1. & \quad (3) \end{aligned}$$

a) We first use *any* two of the three equations to get an equation in two variables. In this case, let's use equations (1) and (2) and add to eliminate z:

$$\begin{aligned} x + y + z &= 4 & \quad (1)\\ x - 2y - z &= 1 & \quad (2)\\ \hline 2x - y \quad &= 5. & \quad (4) \quad \text{Adding to eliminate } z \end{aligned}$$

b) We use a *different* pair of equations and eliminate the **same variable** that we did in part (a). Let's use equations (1) and (3) and again eliminate z.

Caution!

A common error is to eliminate a different variable the second time.

$$\begin{aligned} x + y + z &= 4, & \quad (1)\\ 2x - y - 2z &= -1; & \quad (3) \end{aligned}$$

$$\begin{aligned} 2x + 2y + 2z &= 8 & \quad & \text{Multiplying equation (1) by 2}\\ 2x - y - 2z &= -1 & \quad (3) &\\ \hline 4x + y \quad &= 7 & \quad (5) & \quad \text{Adding to eliminate } z \end{aligned}$$

c) Now we solve the resulting system of equations, (4) and (5). That solution will give us two of the numbers. Note that we now have two equations in two variables. Had we eliminated two *different* variables in parts (a) and (b), this would not be the case.

$$\begin{aligned} 2x - y &= 5 & \quad (4)\\ 4x + y &= 7 & \quad (5)\\ \hline 6x \quad &= 12 & \quad \text{Adding}\\ x &= 2 & \end{aligned}$$

*Other methods for solving systems of equations are considered in Appendixes I and J.

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1. Solve. Don't forget to check.

$$\begin{aligned} 4x - y + z &= 6, \\ -3x + 2y - z &= -3, \\ 2x + y + 2z &= 3 \end{aligned}$$

We can use either equation (4) or (5) to find y. We choose equation (5):

$$\begin{aligned} 4x + y &= 7 && \textbf{(5)} \\ 4(2) + y &= 7 && \text{Substituting 2 for } x \\ 8 + y &= 7 \\ y &= -1. \end{aligned}$$

d) We now have $x = 2$ and $y = -1$. To find the value for z, we use any of the original three equations and substitute to find the third number, z. Let's use equation (1) and substitute our two numbers in it:

$$\begin{aligned} x + y + z &= 4 && \textbf{(1)} \\ 2 + (-1) + z &= 4 && \text{Substituting 2 for } x \text{ and } -1 \text{ for } y \\ \left.\begin{aligned} 1 + z &= 4 \\ z &= 3. \end{aligned}\right\} && && \text{Solving for } z \end{aligned}$$

We have obtained the ordered triple $(2, -1, 3)$. We check as follows, substituting $(2, -1, 3)$ into each of the three equations using alphabetical order.

Check:

$$\begin{array}{r|l} x + y + z = & 4 \\ \hline 2 + (-1) + 3 \ ? & 4 \\ 4 & \end{array} \quad \text{TRUE}$$

$$\begin{array}{r|l} x - 2y - z = & 1 \\ \hline 2 - 2(-1) - 3 \ ? & 1 \\ 2 + 2 - 3 & \\ 1 & \end{array} \quad \text{TRUE}$$

$$\begin{array}{r|l} 2x - y - 2z = & -1 \\ \hline 2(2) - (-1) - 2 \cdot 3 \ ? & -1 \\ 4 + 1 - 6 & \\ -1 & \end{array} \quad \text{TRUE}$$

The triple $(2, -1, 3)$ checks and is the solution.

To use the elimination method to solve systems of three equations:

1. Write all equations in the standard form $Ax + By + Cz = D$.
2. Clear any decimals or fractions.
3. Choose a variable to eliminate. Then use *any* two of the three equations to eliminate that variable, getting an equation in two variables.
4. Next, use a different pair of equations and get another equation in *the same two variables*. That is, eliminate the same variable that you did in step (3).
5. Solve the resulting system (pair) of equations. That will give two of the numbers.
6. Then use any of the original three equations to find the third number.

Do Exercise 1.

Answer on page A-36

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EXAMPLE 2 Solve this system:

$$\begin{aligned} 4x - 2y - 3z &= 5, && \textbf{(1)} \\ -8x - y + z &= -5, && \textbf{(2)} \\ 2x + y + 2z &= 5. && \textbf{(3)} \end{aligned}$$

a) The equations are in standard form and do not contain decimals or fractions.

b) We decide to eliminate the variable y since the y-terms are opposites in equations (2) and (3). We add:

$$\begin{aligned} -8x - y + z &= -5 && \textbf{(2)} \\ 2x + y + 2z &= 5 && \textbf{(3)} \\ \hline -6x \qquad + 3z &= 0. && \textbf{(4)} \quad \text{Adding} \end{aligned}$$

c) We use another pair of equations to get an equation in the same two variables, x and z. That is, we eliminate the same variable y that we did in step (b). We use equations (1) and (3) and eliminate y:

$$\begin{aligned} 4x - 2y - 3z &= 5, && \textbf{(1)} \\ 2x + y + 2z &= 5; && \textbf{(3)} \end{aligned}$$

$$\begin{aligned} 4x - 2y - 3z &= 5 && \textbf{(1)} \\ 4x + 2y + 4z &= 10 && \text{Multiplying equation (3) by 2} \\ \hline 8x \qquad + z &= 15. && \textbf{(5)} \quad \text{Adding} \end{aligned}$$

d) Now we solve the resulting system of equations (4) and (5). That will give us two of the numbers:

$$\begin{aligned} -6x + 3z &= 0, && \textbf{(4)} \\ 8x + z &= 15. && \textbf{(5)} \end{aligned}$$

We multiply equation (5) by -3. (We could also have multiplied equation (4) by $-\frac{1}{3}$.)

$$\begin{aligned} -6x + 3z &= 0 && \textbf{(4)} \\ -24x - 3z &= -45 && \text{Multiplying equation (5) by } -3 \\ \hline -30x \qquad &= -45 && \text{Adding} \\ x &= \tfrac{-45}{-30} = \tfrac{3}{2} \end{aligned}$$

We now use equation (5) to find z:

$$\begin{aligned} 8x + z &= 15 && \textbf{(5)} \\ 8\left(\tfrac{3}{2}\right) + z &= 15 && \text{Substituting } \tfrac{3}{2} \text{ for } x \\ \left.\begin{aligned} 12 + z &= 15 \\ z &= 3. \end{aligned}\right\} & && \text{Solving for } z \end{aligned}$$

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2. Solve. Don't forget to check.

$$\begin{aligned} 2x + y - 4z &= 0, \\ x - y + 2z &= 5, \\ 3x + 2y + 2z &= 3 \end{aligned}$$

e) Next, we use any of the original equations and substitute to find the third number, y. We choose equation (3) since the coefficient of y there is 1:

$$\begin{aligned} 2x + y + 2z &= 5 && \textbf{(3)} \\ 2\left(\tfrac{3}{2}\right) + y + 2(3) &= 5 && \text{Substituting } \tfrac{3}{2} \text{ for } x \text{ and 3 for } z \\ \left.\begin{aligned} 3 + y + 6 &= 5 \\ y + 9 &= 5 \\ y &= -4. \end{aligned}\right\} & && \text{Solving for } y \end{aligned}$$

The solution is $\left(\frac{3}{2}, -4, 3\right)$. The check is as follows.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{4x - 2y - 3z = 5} \\ \hline 4 \cdot \frac{3}{2} - 2(-4) - 3(3) & ?\ 5 \\ 6 + 8 - 9 & \\ 5 & \end{array} \quad \text{TRUE}$$

$$\begin{array}{r|l} \multicolumn{2}{c}{-8x - y + z = -5} \\ \hline -8 \cdot \frac{3}{2} - (-4) + 3 & ?\ -5 \\ -12 + 4 + 3 & \\ -5 & \end{array} \quad \text{TRUE}$$

$$\begin{array}{r|l} \multicolumn{2}{c}{2x + y + 2z = 5} \\ \hline 2 \cdot \frac{3}{2} + (-4) + 2(3) & ?\ 5 \\ 3 - 4 + 6 & \\ 5 & \end{array} \quad \text{TRUE}$$

Do Exercise 2.

In Example 3, two of the equations have a missing variable.

EXAMPLE 3 Solve this system:

$$\begin{aligned} x + y + z &= 180, && \textbf{(1)} \\ x \qquad - z &= -70, && \textbf{(2)} \\ 2y - z &= 0. && \textbf{(3)} \end{aligned}$$

We note that there is no y in equation (2). In order to have a system of two equations in the variables x and z, we need to find another equation without a y. We use equations (1) and (3) to eliminate y:

$$\begin{aligned} x + y + z &= 180, && \textbf{(1)} \\ 2y - z &= 0; && \textbf{(3)} \end{aligned}$$

$$\begin{aligned} -2x - 2y - 2z &= -360 && \text{Multiplying equation (1) by } -2 \\ 2y - z &= 0 && \textbf{(3)} \\ \hline -2x \qquad - 3z &= -360. && \textbf{(4)} \quad \text{Adding} \end{aligned}$$

Now we solve the resulting system of equations (2) and (4):

$$\begin{aligned} x - z &= -70, && \textbf{(2)} \\ -2x - 3z &= -360; && \textbf{(4)} \end{aligned}$$

$$\begin{aligned} 2x - 2z &= -140 && \text{Multiplying equation (2) by 2} \\ -2x - 3z &= -360 && \textbf{(4)} \\ \hline -5z &= -500 && \text{Adding} \\ z &= 100. \end{aligned}$$

Answer on page A-36

To find x, we substitute 100 for z in equation (2) and solve for x:

$$\begin{aligned} x - z &= -70 \\ x - 100 &= -70 \\ x &= 30. \end{aligned}$$

To find y, we substitute 100 for z in equation (3) and solve for y:

$$\begin{aligned} 2y - z &= 0 \\ 2y - 100 &= 0 \\ 2y &= 100 \\ y &= 50. \end{aligned}$$

The triple (30, 50, 100) is the solution. The check is left to the student.

Do Exercise 3.

It is possible for a system of three equations to have no solution, that is, to be inconsistent. An example is the system

$$\begin{aligned} x + y + z &= 14, \\ x + y + z &= 11, \\ 2x - 3y + 4z &= -3. \end{aligned}$$

Note the first two equations. It is not possible for a sum of three numbers to be both 14 and 11. Thus the system has no solution. We will not consider such systems here, nor will we consider systems with infinitely many solutions, which also exist.

3. Solve. Don't forget to check.

$$\begin{aligned} x + y + z &= 100, \\ x - y &= -10, \\ x - z &= -30 \end{aligned}$$

Answer on page A-36

8.5 EXERCISE SET

For Extra Help

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a Solve.

1. $x + y + z = 2,$
$2x - y + 5z = -5,$
$-x + 2y + 2z = 1$

2. $2x - y - 4z = -12,$
$2x + y + z = 1,$
$x + 2y + 4z = 10$

3. $2x - y + z = 5,$
$6x + 3y - 2z = 10,$
$x - 2y + 3z = 5$

4. $x - y + z = 4,$
$3x + 2y + 3z = 7,$
$2x + 9y + 6z = 5$

5. $2x - 3y + z = 5,$
$x + 3y + 8z = 22,$
$3x - y + 2z = 12$

6. $6x - 4y + 5z = 31,$
$5x + 2y + 2z = 13,$
$x + y + z = 2$

7. $3a - 2b + 7c = 13,$
$a + 8b - 6c = -47,$
$7a - 9b - 9c = -3$

8. $x + y + z = 0,$
$2x + 3y + 2z = -3,$
$-x + 2y - 3z = -1$

9. $2x + 3y + z = 17,$
$x - 3y + 2z = -8,$
$5x - 2y + 3z = 5$

10. $2x + y - 3z = -4,$
$4x - 2y + z = 9,$
$3x + 5y - 2z = 5$

11. $2x + y + z = -2,$
$2x - y + 3z = 6,$
$3x - 5y + 4z = 7$

12. $2x + y + 2z = 11,$
$3x + 2y + 2z = 8,$
$x + 4y + 3z = 0$

13. $x - y + z = 4,$
$5x + 2y - 3z = 2,$
$3x - 7y + 4z = 8$

14. $2x + y + 2z = 3,$
$x + 6y + 3z = 4,$
$3x - 2y + z = 0$

15. $4x - y - z = 4,$
$2x + y + z = -1,$
$6x - 3y - 2z = 3$

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16. $\begin{aligned} a + 2b + c &= 1, \\ 7a + 3b - c &= -2, \\ a + 5b + 3c &= 2 \end{aligned}$

17. $\begin{aligned} 2r + 3s + 12t &= 4, \\ 4r - 6s + 6t &= 1, \\ r + s + t &= 1 \end{aligned}$

18. $\begin{aligned} 10x + 6y + z &= 7, \\ 5x - 9y - 2z &= 3, \\ 15x - 12y + 2z &= -5 \end{aligned}$

19. $\begin{aligned} 4a + 9b &= 8, \\ 8a + 6c &= -1, \\ 6b + 6c &= -1 \end{aligned}$

20. $\begin{aligned} 3p + 2r &= 11, \\ q - 7r &= 4, \\ p - 6q &= 1 \end{aligned}$

21. $\begin{aligned} x + y + z &= 57, \\ -2x + y &= 3, \\ x - z &= 6 \end{aligned}$

22. $\begin{aligned} x + y + z &= 105, \\ 10y - z &= 11, \\ 2x - 3y &= 7 \end{aligned}$

23. $\begin{aligned} r + s &= 5, \\ 3s + 2t &= -1, \\ 4r + t &= 14 \end{aligned}$

24. $\begin{aligned} a - 5c &= 17, \\ b + 2c &= -1, \\ 4a - b - 3c &= 12 \end{aligned}$

25. DW Explain a procedure that could be used to solve a system of four equations in four variables.

26. DW Is it possible for a system of three equations to have exactly two ordered triples in its solution set? Why or why not?

SKILL MAINTENANCE

Solve for the indicated letter. [2.4b]

27. $F = 3ab$, for a

28. $Q = 4(a + b)$, for a

29. $F = \frac{1}{2}t(c - d)$, for c

30. $F = \frac{1}{2}t(c - d)$, for d

31. $Ax - By = c$, for y

32. $Ax + By = c$, for y

Find the slope and the y-intercept. [7.3b]

33. $y = -\frac{2}{3}x - \frac{5}{4}$

34. $y = 5 - 4x$

35. $2x - 5y = 10$

36. $7x - 6.4y = 20$

SYNTHESIS

Solve.

37. $\begin{aligned} w + x + y + z &= 2, \\ w + 2x + 2y + 4z &= 1, \\ w - x + y + z &= 6, \\ w - 3x - y + z &= 2 \end{aligned}$

38. $\begin{aligned} w + x - y + z &= 0, \\ w - 2x - 2y - z &= -5, \\ w - 3x - y + z &= 4, \\ 2w - x - y + 3z &= 7 \end{aligned}$

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Objective

a Solve applied problems using systems of three equations.

1. **Triangle Measures.** One angle of a triangle is twice as large as a second angle. The remaining angle is 20° greater than the first angle. Find the measure of each angle.

Answer on page A-36

8.6 SOLVING APPLIED PROBLEMS: THREE EQUATIONS

a Using Systems of Three Equations

Solving systems of three or more equations is important in many applications occurring in the natural and social sciences, business, and engineering.

EXAMPLE 1 *Architecture.* In a triangular cross-section of a roof, the largest angle is 70° greater than the smallest angle. The largest angle is twice as large as the remaining angle. Find the measure of each angle.

1. **Familiarize.** We first make a drawing. Since we do not know the size of any angle, we use x, y, and z for the measures of the angles. We let $x =$ the smallest angle, $z =$ the largest angle, and $y =$ the remaining angle.

2. **Translate.** In order to translate the problem, we need to make use of a geometric fact—that is, the sum of the measures of the angles of a triangle is 180°. This fact about triangles gives us one equation:

$$x + y + z = 180.$$

There are two statements in the problem that we can translate directly.

$$\underbrace{\text{The largest angle}}_{z}\;\underbrace{\text{is}}_{=}\;\underbrace{70^\circ}_{70}\;\underbrace{\text{greater than}}_{+}\;\underbrace{\text{the smallest angle.}}_{x}$$

$$\underbrace{\text{The largest angle}}_{z}\;\underbrace{\text{is}}_{=}\;\underbrace{\text{twice as large as the remaining angle.}}_{2y}$$

We now have a system of three equations:

$$\begin{array}{lcl} x + y + z = 180, & & x + y + z = 180, \\ x + 70 = z, & \text{or} & x \qquad - z = -70, \\ 2y = z; & & \quad 2y - z = 0. \end{array}$$

3. **Solve.** The system was solved in Example 3 of Section 8.5. The solution is $(30, 50, 100)$.

4. **Check.** The sum of the numbers is 180. The largest angle measures 100° and the smallest measures 30°. The largest angle is 70° greater than the smallest. The remaining angle measures 50°. The largest angle is twice as large as the remaining angle. We do have an answer to the problem.

5. **State.** The measures of the angles of the triangle are 30°, 50°, and 100°.

Do Exercise 1.

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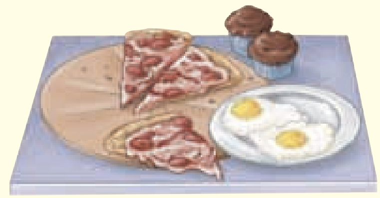
EXAMPLE 2 *Cholesterol Levels.* Americans have become very conscious of their cholesterol levels. Recent studies indicate that a child's intake of cholesterol should be no more than 300 mg per day. By eating 1 egg, 1 cupcake, and 1 slice of pizza, a child consumes 302 mg of cholesterol. If the child eats 2 cupcakes and 3 slices of pizza, he or she takes in 65 mg of cholesterol. By eating 2 eggs and 1 cupcake, a child consumes 567 mg of cholesterol. How much cholesterol is in each item?



1. **Familiarize.** After we have read the problem a few times, it becomes clear that an egg contains considerably more cholesterol than the other foods. Let's guess that one egg contains 200 mg of cholesterol and one cupcake contains 50 mg. Because of the third sentence in the problem, it would follow that a slice of pizza contains 52 mg of cholesterol since $200 + 50 + 52 = 302$.

   To see if our guess satisfies the other statements in the problem, we find the amount of cholesterol that 2 cupcakes and 3 slices of pizza would contain: $2 \cdot 50 + 3 \cdot 52 = 256$. Since this does not match the 65 mg listed in the fourth sentence of the problem, our guess was incorrect. Rather than guess again, we examine how we checked our guess and let e, c, and s = the number of milligrams of cholesterol in an egg, a cupcake, and a slice of pizza, respectively.

2. **Translate.** By rewording some of the sentences in the problem, we can translate it into three equations.

$$\underbrace{\text{The amount of cholesterol in 1 egg}}_{e}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{the amount of cholesterol in 1 cupcake}}_{c}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{the amount of cholesterol in 1 slice of pizza}}_{s}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{302 mg.}}_{302}$$

$$\underbrace{\text{The amount of cholesterol in 2 cupcakes}}_{2c}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{the amount of cholesterol in 3 slices of pizza}}_{3s}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{65 mg.}}_{65}$$

$$\underbrace{\text{The amount of cholesterol in 2 eggs}}_{2e}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{the amount of cholesterol in 1 cupcake}}_{c}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{567 mg.}}_{567}$$

We now have a system of three equations:

$$\begin{aligned} e + c + s &= 302, & \textbf{(1)}\\ 2c + 3s &= 65, & \textbf{(2)}\\ 2e + c &= 567. & \textbf{(3)} \end{aligned}$$

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2. Client Investments. Kaufman Financial Corporation makes investments for corporate clients. One year, a client receives $1620 in simple interest from three investments that total $25,000. Part is invested at 5%, part at 6%, and part at 7%. There is $11,000 more invested at 7% than at 6%. How much was invested at each rate?

3. Solve. To solve, we first note that the variable e does not appear in equation (2). In order to have a system of two equations in the variables c and s, we need to find another equation without the variable e. We use equations (1) and (3) to eliminate e:

$$\begin{aligned} e + c + s &= 302, \quad &\textbf{(1)} \\ 2e + c \qquad &= 567; \quad &\textbf{(3)} \end{aligned}$$

$$\begin{array}{rll} -2e - 2c - 2s = -604 & & \text{Multiplying equation (1) by } -2 \\ \underline{2e + c \phantom{{}-2s} = 567} & \textbf{(3)} & \\ -c - 2s = -37. & \textbf{(4)} & \text{Adding} \end{array}$$

Next, we solve the resulting system of equations (2) and (4):

$$\begin{aligned} 2c + 3s &= 65, \quad &\textbf{(2)} \\ -c - 2s &= -37; \quad &\textbf{(4)} \end{aligned}$$

$$\begin{array}{rl} 2c + 3s = 65 & \textbf{(2)} \\ \underline{-2c - 4s = -74} & \text{Multiplying equation (4) by 2} \\ -s = -9 & \text{Adding} \\ s = 9. & \end{array}$$

To find c, we substitute 9 for s in equation (4) and solve for c:

$$\begin{aligned} -c - 2s &= -37 \quad &\textbf{(4)} \\ -c - 2(9) &= -37 \quad &\text{Substituting} \\ -c - 18 &= -37 \\ -c &= -19 \\ c &= 19. \end{aligned}$$

To find e, we substitute 19 for c in equation (3) and solve for e:

$$\begin{aligned} 2e + c &= 567 \quad &\textbf{(3)} \\ 2e + 19 &= 567 \quad &\text{Substituting} \\ 2e &= 548 \\ e &= 274. \end{aligned}$$

The solution is $c = 19$, $e = 274$, $s = 9$, or $(19, 274, 9)$.

4. Check. The sum of 19, 274, and 9 is 302 so the total cholesterol in 1 cupcake, 1 egg, and 1 slice of pizza checks. Two cupcakes and three slices of pizza would contain $2 \cdot 19 + 3 \cdot 9$, or 65 mg, while two eggs and one cupcake would contain $2 \cdot 274 + 19$, or 567 mg of cholesterol. The answer checks.

5. State. A cupcake contains 19 mg of cholesterol, an egg contains 274 mg of cholesterol, and a slice of pizza contains 9 mg of cholesterol.

Do Exercise 2.

Answer on page A-36

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For Extra Help

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a Solve.

1. *Low-Fat Fruit Drinks.* Smoothie King®, Nutritional Lifestyle Center ™, recently sold 20-oz low-fat fruit Smoothies for \$3.25, 32-oz Smoothies for \$4.75, and 40-oz Smoothies for \$5.75. One hot summer afternoon, Jake sold 34 Smoothies for a total of \$153. The number of 20-oz and 40-oz Smoothies, combined, was 4 more than the number of 32-oz Smoothies. How many of each size were sold?

   Source: Smoothie King®

2. *Cappuccinos.* Starbucks® sells cappuccinos in three sizes: tall for \$2.65, grande for \$3.20, and vente for \$3.50. After an outdoor rally for the college football team, Bryce served 50 cappuccinos. The number of tall and vente cappuccinos, combined, was 2 fewer than the number of grande cappuccinos. If he collected a total of \$157, how many cappuccinos of each size did he serve?

   Source: Starbucks® Corporation

3. *Triangle Measures.* In triangle ABC, the measure of angle B is three times that of angle A. The measure of angle C is 20° more than that of angle A. Find the measure of each angle.

4. *Triangle Measures.* In triangle ABC, the measure of angle B is twice the measure of angle A. The measure of angle C is 80° more than that of angle A. Find the measure of each angle.

5. The sum of three numbers is 55. The difference of the largest and the smallest is 49, and the sum of the two smaller is 13. Find the numbers.

6. The sum of three numbers is −30. The largest minus twice the smallest is 45, and the largest is 20 more than the middle number. Find the numbers.

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7. *Automobile Pricing.* A recent basic model of a particular automobile had a price of $12,685. The basic model with the added features of automatic transmission and power door locks was $14,070. The basic model with air conditioning (AC) and power door locks was $13,580. The basic model with AC and automatic transmission was $13,925. What was the individual cost of each of the three options?

8. *Telemarketing.* Sven, Tillie, and Isaiah can process 740 telephone orders per day. Sven and Tillie together can process 470 orders, while Tillie and Isaiah together can process 520 orders per day. How many orders can each person process alone?

9. *Lens Production.* When Sight-Rite's three polishing machines, A, B, and C, are all working, 5700 lenses can be polished in one week. When only A and B are working, 3400 lenses can be polished in one week. When only B and C are working, 4200 lenses can be polished in one week. How many lenses can be polished in a week by each machine alone?

10. *Welding Rates.* Elrod, Dot, and Wendy can weld 74 linear feet per hour when working together. Elrod and Dot together can weld 44 linear feet per hour, while Elrod and Wendy can weld 50 linear feet per hour. How many linear feet per hour can each weld alone?

11. *Investments.* A business class divided an imaginary investment of $80,000 among three mutual funds. The first fund grew by 10%, the second by 6%, and the third by 15%. Total earnings were $8850. The earnings from the first fund were $750 more than the earnings from the third. How much was invested in each fund?

12. *Advertising.* In a recent year, companies spent a total of $84.8 billion on newspaper, television, and radio ads. The total amount spent on television and radio ads was only $2.6 billion more than the amount spent on newspaper ads alone. The amount spent on newspaper ads was $5.1 billion more than what was spent on television ads. How much was spent on each form of advertising? (*Hint*: Let the variables represent numbers of billions of dollars.)

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13. *Twin Births.* In the United States, the highest incidence of fraternal twin births occurs among Asian–Americans, then African–Americans, and then Caucasians. Of every 15,400 births, the total number of fraternal twin births for all three is 739, where there are 185 more for Asian–Americans than African–Americans and 231 more for Asian–Americans than Caucasians. How many births of fraternal twins are there for each group out of every 15,400 births?

14. *Crying Rate.* The sum of the average number of times a man, a woman, and a one-year-old child cry each month is 71.7. A one-year-old cries 46.4 more times than a man. The average number of times a one-year-old cries per month is 28.3 more than the average number of times combined that a man and a woman cry. What is the average number of times per month that each cries?

15. *Nutrition.* A dietician in a hospital prepares meals under the guidance of a physician. Suppose that for a particular patient a physician prescribes a meal to have 800 calories, 55 g of protein, and 220 mg of vitamin C. The dietician prepares a meal of roast beef, baked potato, and broccoli according to the data in the following table.

	CALORIES	PROTEIN (in grams)	VITAMIN C (in milligrams)
Roast Beef, 3 oz	300	20	0
Baked Potato	100	5	20
Broccoli, 156 g	50	5	100

How many servings of each food are needed in order to satisfy the doctor's orders?

16. *Nutrition.* Repeat Exercise 15 but replace the broccoli with asparagus, for which one 180-g serving contains 50 calories, 5 g of protein, and 44 mg of vitamin C. Which meal would you prefer eating?

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17. *Golf.* On an 18-hole golf course, there are par-3 holes, par-4 holes, and par-5 holes. A golfer who shoots par on every hole has a total of 70. There are twice as many par-4 holes as there are par-5 holes. How many of each type of hole are there on the golf course?

18. *Golf.* On an 18-hole golf course, there are par-3 holes, par-4 holes, and par-5 holes. A golfer who shoots par on every hole has a total of 72. The sum of the number of par-3 holes and the number of par-5 holes is 8. How many of each type of hole are there on the golf course?

19. *Basketball Scoring.* The New York Knicks recently scored a total of 92 points on a combination of 2-point field goals, 3-point field goals, and 1-point foul shots. Altogether, the Knicks made 50 baskets and 19 more 2-pointers than foul shots. How many shots of each kind were made?

20. *History.* Find the year in which the first U.S. transcontinental railroad was completed. The following are some facts about the number. The sum of the digits in the year is 24. The ones digit is 1 more than the hundreds digit. Both the tens and the ones digits are multiples of 3.

21. D_W Exercise 10 can be solved mentally after a careful reading of the problem. How is this possible?

22. D_W *Ticket Revenue.* A pops-concert audience of 100 people consists of adults, senior citizens, and children. The ticket prices are $10 each for adults, $3 each for senior citizens, and $0.50 each for children. The total amount of money taken in is $100. How many adults, senior citizens, and children are in attendance? Does there seem to be some information missing? Do some careful reasoning and explain.

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23. DW Consider Exercise 19. Suppose 92 points were scored with 50 baskets but no foul shots. Would there still be a solution? Why or why not?

24. DW Consider Exercise 1. Suppose Jake collected \$150. Could the problem still be solved? Why or why not?

SKILL MAINTENANCE

VOCABULARY REINFORCEMENT

In each of Exercises 25–32, fill in the blank with the correct term from the given list. Some of the choices may not be used.

25. The expression $x \leq q$ means x is ________________ q. [2.8a]

26. The expression $x \geq q$ means x is ________________ q. [2.8a]

27. The graph of a(n) ________________ equation is a line. [3.2b]

28. When the slope of a line is ________________, the graph of the line slants down from left to right. [3.4a]

29. A(n) ________________ system of equations has at least one solution. [8.1a]

30. Two lines are ________________ if the product of their slopes is -1. [7.4d]

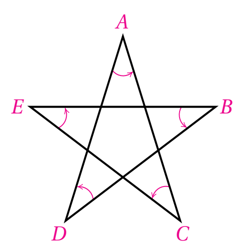
31. The ________________ of the graph of $f(x) = mx + b$ is the point $(0, b)$. [7.3a]

32. When the slope of a line is zero, the graph of the line is ________________. [7.3b]

parallel
perpendicular
consistent
inconsistent
linear
x-intercept
y-intercept
positive
zero
negative
vertical
horizontal
at least
at most

SYNTHESIS

33. Find the sum of the angle measures at the tips of the star in this figure.



34. *Sharing Raffle Tickets.* Hal gives Tom as many raffle tickets as Tom has and Gary as many as Gary has. In like manner, Tom then gives Hal and Gary as many tickets as each then has. Similarly, Gary gives Hal and Tom as many tickets as each then has. If each finally has 40 tickets, with how many tickets does Tom begin?

35. *Digits.* Find a three-digit positive integer such that the sum of all three digits is 14, the tens digit is 2 more than the ones digit, and if the digits are reversed, the number is unchanged.

36. *Ages.* Tammy's age is the sum of the ages of Carmen and Dennis. Carmen's age is 2 more than the sum of the ages of Dennis and Mark. Dennis's age is four times Mark's age. The sum of all four ages is 42. How old is Tammy?

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Introductory and Intermediate Algebra, Third Edition, by Marvin L.Bittinger and Judith A.Beecher.Published by Addison Wesley.

8.7 BUSINESS AND ECONOMICS APPLICATIONS

Objectives

a Given total-cost and total-revenue functions, find the total-profit function and the break-even point.

b Given supply and demand functions, find the equilibrium point.

a Break-Even Analysis

When a company manufactures x units of a product, it invests money. This is **total cost** and can be thought of as a function C, where $C(x)$ is the total cost of producing x units. When the company sells x units of the product, it takes in money. This is **total revenue** and can be thought of as a function R, where $R(x)$ is the total revenue from the sale of x units. **Total profit** is the money taken in less the money spent, or total revenue minus total cost. Total profit from the production and sale of x units is a function P given by

Profit = Revenue − Cost, or $P(x) = R(x) - C(x)$.

If $R(x)$ is greater than $C(x)$, the company has a profit. If $C(x)$ is greater than $R(x)$, the company has a loss. When $R(x) = C(x)$, the company breaks even.

There are two kinds of costs. First, there are costs like rent, insurance, machinery, and so on. These costs, which must be paid whether a product is produced or not, are called **fixed costs.** When a product is being produced, there are costs for labor, materials, marketing, and so on. These are called **variable costs,** because they vary according to the amount of the product being produced. The sum of the fixed costs and the variable costs gives the **total cost** of producing a product.

EXAMPLE 1 *Manufacturing Radios.* Ergs, Inc., is planning to make a new kind of radio. Fixed costs will be \$90,000, and it will cost \$15 to produce each radio (variable costs). Each radio sells for \$26.

a) Find the total cost $C(x)$ of producing x radios.

b) Find the total revenue $R(x)$ from the sale of x radios.

c) Find the total profit $P(x)$ from the production and sale of x radios.

d) What profit or loss will the company realize from the production and sale of 3000 radios? of 14,000 radios?

e) Graph the total-cost, total-revenue, and total-profit functions using the same set of axes. Determine the break-even point.

a) Total cost is given by

$$C(x) = (\text{Fixed costs}) \text{ plus } (\text{Variable costs}),$$

$$\text{or} \quad C(x) = 90{,}000 + 15x,$$

where x is the number of radios produced.

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b) Total revenue is given by

$R(x) = 26x.$ $26 times the number of radios sold. We assume that every radio produced is sold.

c) Total profit is given by

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 26x - (90{,}000 + 15x) \\ &= 11x - 90{,}000. \end{aligned}$$

d) Profits will be

$$P(3000) = 11 \cdot 3000 - 90{,}000 = -\$57{,}000$$

when 3000 radios are produced and sold, and

$$P(14{,}000) = 11 \cdot 14{,}000 - 90{,}000 = \$64{,}000$$

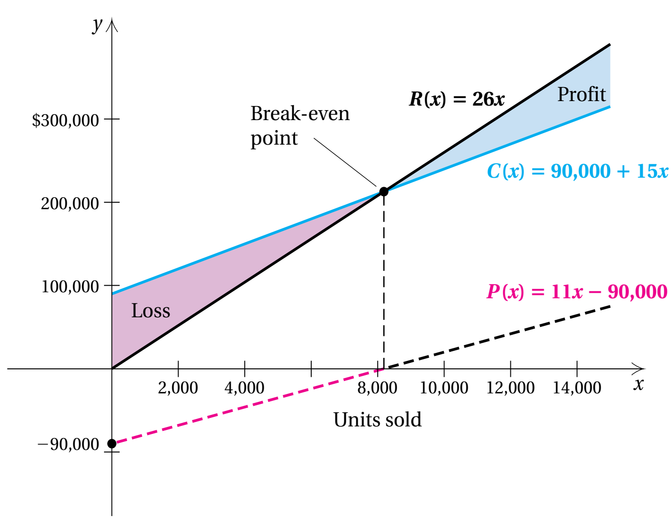
when 14,000 radios are produced and sold. Thus the company loses $57,000 if only 3000 radios are sold, but makes $64,000 if 14,000 are sold.

e) The graphs of each of the three functions are shown below:

$$\begin{aligned} C(x) &= 90{,}000 + 15x, && \textbf{(1)} \\ R(x) &= 26x, && \textbf{(2)} \\ P(x) &= 11x - 90{,}000. && \textbf{(3)} \end{aligned}$$

$R(x)$, $C(x)$, and $P(x)$ are all in dollars.

Equation (2) has a graph that goes through the origin and has a slope of 26. Equation (1) has an intercept on the y-axis of 90,000 and has a slope of 15. Equation (3) has an intercept on the y-axis of $-90{,}000$ and has a slope of 11. It is shown by the dashed line. The red dashed line shows a "negative" profit, which is a loss. (That is what is known as "being in the red.") The black dashed line shows a "positive" profit, or gain. (That is what is known as "being in the black.")



Profits occur when the revenue is greater than the cost. Losses occur when the revenue is less than the cost. The **break-even point** occurs where the graphs of R and C cross. Thus to find the break-even point, we solve a system:

$$\begin{aligned} C(x) &= 90{,}000 + 15x, \\ R(x) &= 26x. \end{aligned}$$

1. Manufacturing Radios. Refer to Example 1. Suppose that fixed costs are $80,000, and it costs $20 to produce each radio. Each radio sells for $36.

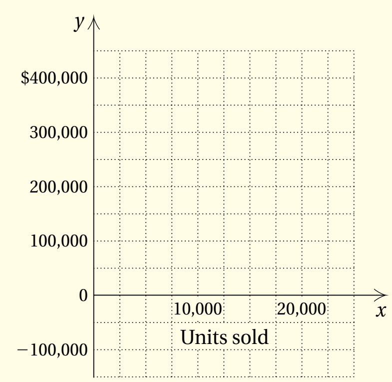
a) Find the total cost $C(x)$ of producing x radios.

b) Find the total revenue $R(x)$ from the sale of x radios.

c) Find the total profit $P(x)$ from the production and sale of x radios.

d) What profit or loss will the company realize from the production and sale of 4000 radios? of 16,000 radios?

e) Graph the total-cost, total-revenue, and total-profit functions using the same set of axes. Determine the break-even point.



Answers on page A-37

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Since both revenue and cost are in *dollars* and they are equal at the break-even point, the system can be rewritten as

$$d = 90{,}000 + 15x, \quad (1)$$
$$d = 26x \quad (2)$$

and solved using substitution:

$$26x = 90{,}000 + 15x \qquad \text{Substituting } 26x \text{ for } d \text{ in equation (1)}$$
$$11x = 90{,}000$$
$$x \approx 8181.8.$$

The firm will break even if it produces and sells about 8182 radios (8181 will yield a tiny loss and 8182 a tiny gain), and takes in a total of $R(8182) = 26 \cdot 8182 = \$212{,}732$ in revenue. Note that the x-coordinate of the break-even point is also the x-coordinate of the x-intercept of the profit function. It can also be found by solving $P(x) = 0$.

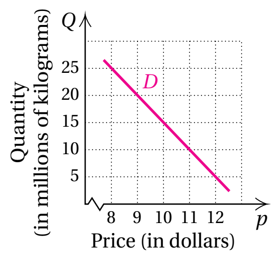
Do Exercise 1 on the preceding page.

b Supply and Demand

As the price of coffee varies, the amount sold varies. The table and the graph below both show that consumer demand goes down as the price goes up and the demand goes up as the price goes down.

DEMAND FUNCTION, *D*

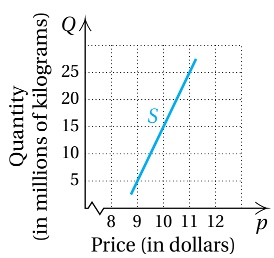
PRICE, *p*, PER KILOGRAM	QUANTITY, *D*(*p*) (in millions of kilograms)
$ 8.00	25
9.00	20
10.00	15
11.00	10
12.00	5



As the price of coffee varies, the amount available varies. The table and the graph below both show that sellers will supply less as the price goes down, but will supply more as the price goes up.

SUPPLY FUNCTION, *S*

PRICE, *p*, PER KILOGRAM	QUANTITY, *S*(*p*) (in millions of kilograms)
$ 9.00	5
9.50	10
10.00	15
10.50	20
11.00	25



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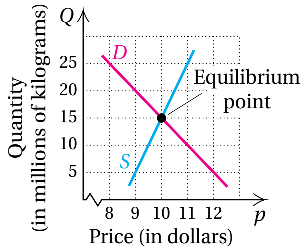
Let's look at the above graphs together. We see that as price increases, demand decreases. As price increases, supply increases. The point of intersection of the demand and supply functions is called the **equilibrium point.** At the equilibrium point, the amount that the seller will supply is the same amount that the consumer will buy. The situation is analogous to a buyer and a seller negotiating the price of an item. The equilibrium point is the price and quantity on which they finally agree.



Any ordered pair of coordinates from the graph is (price, quantity), because the horizontal axis is the price axis and the vertical axis is the quantity axis. If D is a demand function and S is a supply function, then the equilibrium point is where demand equals supply:

$$D(p) = S(p).$$

EXAMPLE 2 Find the equilibrium point for the following demand and supply functions:

$$D(p) = 1000 - 60p, \quad \textbf{(1)}$$
$$S(p) = 200 + 4p. \quad \textbf{(2)}$$

Since both demand and supply are *quantities* and they are equal at the equilibrium point, we rewrite the system as

$$q = 1000 - 60p, \quad \textbf{(1)}$$
$$q = 200 + 4p. \quad \textbf{(2)}$$

We substitute $200 + 4p$ for q in equation (1) and solve:

$$\begin{aligned} 200 + 4p &= 1000 - 60p \\ 200 + 64p &= 1000 && \text{Adding } 60p \text{ on both sides} \\ 64p &= 800 && \text{Subtracting 200 on both sides} \\ p &= \frac{800}{64} = 12.5. \end{aligned}$$

Thus the equilibrium price is \$12.50 per unit.

To find the equilibrium quantity, we substitute \$12.50 into either $D(p)$ or $S(p)$. We use $S(p)$:

$$\begin{aligned} S(12.5) &= 200 + 4(12.5) \\ &= 200 + 50 = 250. \end{aligned}$$

Thus the equilibrium quantity is 250 units, and the equilibrium point is (\$12.50, 250).

Do Exercise 2.

2. Find the equilibrium point for the following supply and demand functions:

$$D(p) = 1000 - 46p,$$
$$S(p) = 300 + 4p.$$

Answer on page A-37

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a For each of the following pairs of total-cost and total-revenue functions, find **(a)** the total-profit function and **(b)** the break-even point.

1. $C(x) = 25x + 270{,}000$; $R(x) = 70x$

2. $C(x) = 45x + 300{,}000$; $R(x) = 65x$

3. $C(x) = 10x + 120{,}000$; $R(x) = 60x$

4. $C(x) = 30x + 49{,}500$; $R(x) = 85x$

5. $C(x) = 20x + 10{,}000$; $R(x) = 100x$

6. $C(x) = 40x + 22{,}500$; $R(x) = 85x$

7. $C(x) = 22x + 16{,}000$; $R(x) = 40x$

8. $C(x) = 15x + 75{,}000$; $R(x) = 55x$

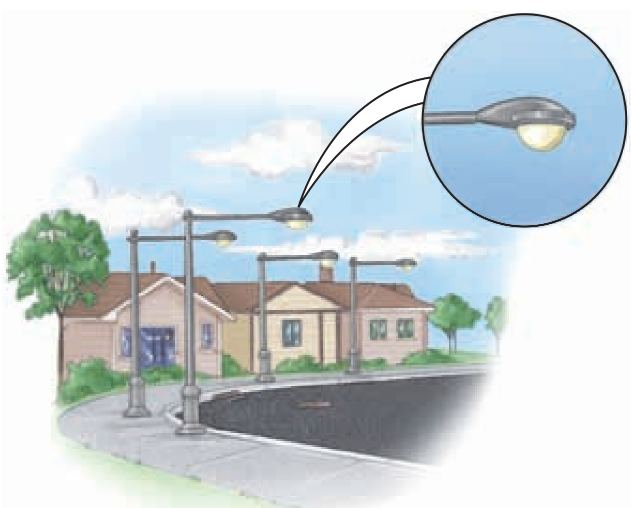
9. $C(x) = 50x + 195{,}000$; $R(x) = 125x$

10. $C(x) = 34x + 928{,}000$; $R(x) = 128x$

Solve.

11. *Manufacturing Lamps.* City Lights is planning to manufacture a new type of lamp. For the first year, the fixed costs for setting up production are \$22,500. The variable costs for producing each lamp are \$40. The revenue from each lamp is \$85. Find the following.

a) The total cost $C(x)$ of producing x lamps
b) The total revenue $R(x)$ from the sale of x lamps
c) The total profit $P(x)$ from the production and sale of x lamps
d) The profit or loss from the production and sale of 3000 lamps; of 400 lamps
e) The break-even point



12. *Computer Manufacturing.* Sky View Electronics is planning to introduce a new line of computers. For the first year, the fixed costs for setting up production are \$125,100. The variable costs for producing each computer are \$750. The revenue from each computer is \$1050. Find the following.

a) The total cost $C(x)$ of producing x computers
b) The total revenue $R(x)$ from the sale of x computers
c) The total profit $P(x)$ from the production and sale of x computers
d) The profit or loss from the production and sale of 400 computers; of 700 computers
e) The break-even point

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Solve.

13. *Manufacturing Caps.* Martina's Custom Printing is planning on adding painter's caps to its product line. For the first year, the fixed costs for setting up production are $16,404. The variable costs for producing a dozen caps are $6.00. The revenue on each dozen caps is $18.00. Find the following.

a) The total cost $C(x)$ of producing x dozen caps
b) The total revenue $R(x)$ from the sale of x dozen caps
c) The total profit $P(x)$ from the production and sale of x dozen caps
d) The profit or loss from the production and sale of 3000 dozen caps; of 1000 dozen caps
e) The break-even point

14. *Sport Coat Production.* Sarducci's is planning a new line of sport coats. For the first year, the fixed costs for setting up production are $10,000. The variable costs for producing each coat are $20. The revenue from each coat is $100. Find the following.

a) The total cost $C(x)$ of producing x coats
b) The total revenue $R(x)$ from the sale of x coats
c) The total profit $P(x)$ from the production and sale of x coats
d) The profit or loss from the production and sale of 2000 coats; of 50 coats
e) The break-even point

b Find the equilibrium point for each of the following pairs of demand and supply functions.

15. $D(p) = 1000 - 10p;$ $S(p) = 230 + p$

16. $D(p) = 2000 - 60p;$ $S(p) = 460 + 94p$

17. $D(p) = 760 - 13p;$ $S(p) = 430 + 2p$

18. $D(p) = 800 - 43p;$ $S(p) = 210 + 16p$

19. $D(p) = 7500 - 25p;$ $S(p) = 6000 + 5p$

20. $D(p) = 8800 - 30p;$ $S(p) = 7000 + 15p$

21. $D(p) = 1600 - 53p;$ $S(p) = 320 + 75p$

22. $D(p) = 5500 - 40p;$ $S(p) = 1000 + 85p$

23. D$_W$ Variable costs and fixed costs are often compared to the slope and the y-intercept, respectively, of an equation of a line. Explain why this analogy is valid.

24. D$_W$ In this section, we examined supply and demand functions for coffee. Does it seem realistic to you for the graph of D to have a constant slope? Why or why not?

SKILL MAINTENANCE

Find the slope and the y-intercept. [7.3b]

25. $5y - 3x = 8$

26. $6x + 7y - 9 = 4$

27. $2y = 3.4x + 98$

28. $\frac{x}{3} + \frac{y}{4} = 1$

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8 Summary and Review

The review that follows is meant to prepare you for a chapter exam. It consists of two parts. The first part, Concept Reinforcement, is designed to increase understanding of the concepts through true/false exercises. The second part is the Review Exercises. These provide practice exercises for the exam, together with references to section objectives so you can go back and review. Before beginning, stop and look back over the skills you have obtained. What skills in mathematics do you have now that you did not have before studying this chapter?

CONCEPT REINFORCEMENT

Determine whether the statement is true or false. Answers are given at the back of the book.

_______ **1.** When $(0, b)$ is a solution of each equation in a system of two equations, the graphs of the two equations have the same y-intercept.

_______ **2.** If, when solving a system of two linear equations in two variables, a false equation is obtained, the system has infinitely many solutions.

_______ **3.** Every system of equations has at least one solution.

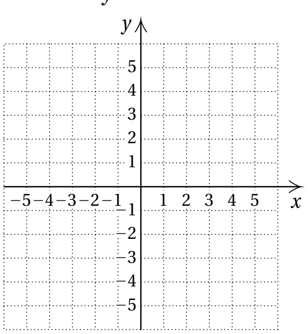
_______ **4.** If the graphs of two linear equations intersect, then the system is consistent.

_______ **5.** The intersection of the graphs of the lines $x = a$ and $y = b$ is (a, b).

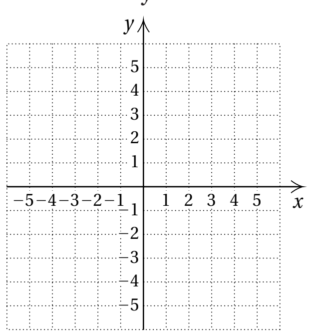
Review Exercises

Solve graphically. Then classify the system as consistent or inconsistent and the equations as dependent or independent. [8.1a]

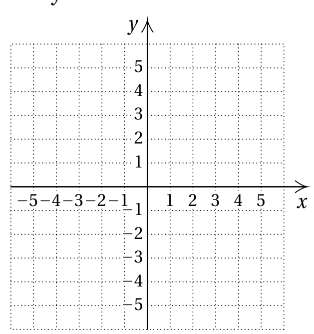
1. $4x - y = -9,$
$x - y = -3$



2. $15x + 10y = -20,$
$3x + 2y = -4$



3. $y - 2x = 4,$
$y - 2x = 5$



Solve by the substitution method. [8.2a]

4. $7x - 4y = 6,$
$y - 3x = -2$

5. $y = x + 2,$
$y - x = 8$

6. $9x - 6y = 2,$
$x = 4y + 5$

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Solve by the elimination method. [8.3a]

7. $8x - 2y = 10,$
$-4y - 3x = -17$

8. $4x - 7y = 18,$
$9x + 14y = 40$

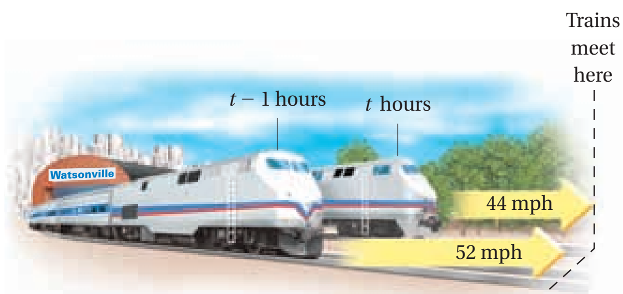
9. $3x - 5y = -4,$
$5x - 3y = 4$

10. $1.5x - 3 = -2y,$
$3x + 4y = 6$

11. *Music Spending.* Sean has $37 to spend. He can spend all of it on two compact discs and a cassette, or he can buy one CD and two cassettes and have $5.00 left over. What is the price of a CD? of a cassette? [8.4a]

12. *Orange Drink Mixtures.* "Orange Thirst" is 15% orange juice and "Quencho" is 5% orange juice. How many liters of each should be combined in order to get 10 L of a mixture that is 10% orange juice? [8.4a]

13. *Train Travel.* A train leaves Watsonville at noon traveling north at 44 mph. One hour later, another train, going 52 mph, travels north on a parallel track. How many hours will the second train travel before it overtakes the first train? [8.4b]



Solve. [8.5a]

14. $x + 2y + z = 10,$
$2x - y + z = 8,$
$3x + y + 4z = 2$

15. $3x + 2y + z = 3,$
$6x - 4y - 2z = -34,$
$-x + 3y - 3z = 14$

16. $2x - 5y - 2z = -4,$
$7x + 2y - 5z = -6,$
$-2x + 3y + 2z = 4$

17. $x + y + 2z = 1,$
$x - y + z = 1,$
$x + 2y + z = 2$

18. *Triangle Measure.* In triangle ABC, the measure of angle A is four times the measure of angle C, and the measure of angle B is 45° more than the measure of angle C. What are the measures of the angles of the triangle? [8.6a]

19. *Money Mixtures.* Elaine has $194, consisting of $20, $5, and $1 bills. The number of $1 bills is 1 less than the total number of $20 and $5 bills. If she has 39 bills in her purse, how many of each denomination does she have? [8.6a]

20. *Bed Manufacturing.* Kregel Furniture is planning to produce a new type of bed. For the first year, the fixed costs for setting up production are $35,000. The variable costs for producing each bed are $175. The revenue from each bed is $300. Find the following. [8.7a]

a) The total cost $C(x)$ of producing x beds
b) The total revenue $R(x)$ from the sale of x beds
c) The total profit from the production and sale of x beds
d) The profit or loss from the production and sale of 1200 beds; of 200 beds
e) The break-even point

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21. Find the equilibrium point for the following demand and supply functions: [8.7b]

$$D(p) = 120 - 13p,$$
$$S(p) = 60 + 7p.$$

22. **DW** Briefly compare the strengths and the weaknesses of the graphical, substitution, and elimination methods as applied to the solution of two equations in two variables. [8.1a], [8.2a], [8.3a]

23. **DW** Explain the advantages of using a system of equations to solve an applied problem. [8.2b], [8.3b], [8.4a, b]

24. *Peanuts.* Help Peppermint Patty with her dilemma. Translate Peppermint Patty's problem to a system of equations. Then solve the problem. [8.3b]

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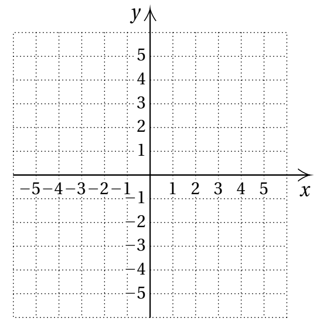
SYNTHESIS

25. Solve graphically:

$$y = x + 2,$$
$$y = x^2 + 2.$$

[8.1a]



26. *Height Estimation in Anthropology.* An anthropologist can use linear functions to estimate the height of a male or female, given the length of certain bones. The *femur* is the large bone from the hip to the knee. Let x = the length of the femur, in centimeters. Then the height, in centimeters, of a male with a femur of length x is given by the function

$$M(x) = 1.88x + 81.31.$$

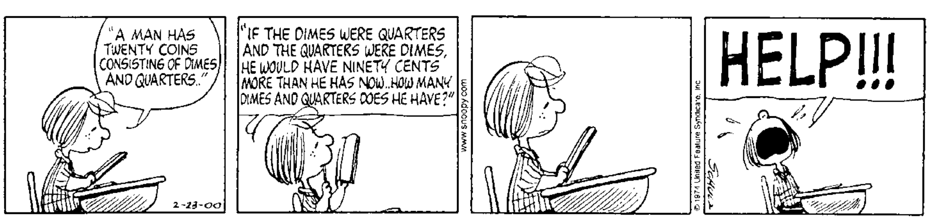
The height, in centimeters, of a female with a femur of length x is given by the function

$$F(x) = 1.95x + 72.85.$$

[7.1b], [8.1a]

A 45-cm femur was uncovered at an anthropological dig.

a) If we assume that it was from a male, how tall was he?
b) If we assume that it was from a female, how tall was she?
c) Graph each equation and find the point of intersection of the graphs of the equations.
d) For what length of a male femur and a female femur, if any, would the height be the same?



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