The Central Police Department had recently been criticized in the local media for not responding to police calls in the downtown area rapidly enough. In several recent cases, alarms had sounded for break-ins, but by the time the police car arrived, the perpetrators had left, and in one instance a store owner had been shot. Sergeant Davies had been assigned by the chief to find a way to determine the optimal patrol area (dimensions) for their cars that would minimize the average time it took to respond to a call in the downtown area.

Sergeant Davies solicited help from Ms. Maris, an analyst in the operations area for the police department. Together they began to work through the problem.

Sergeant Davies noted to Ms. Maris that normal patrol sectors are laid out in rectangles, with each rectangle including a number of city blocks. For illustrative purposes he defined the dimensions of the sector as ***x*** in the horizontal direction and as ***y*** in the vertical direction. He explained to Ms. Maris that cars traveled in straight lines either horizontally or vertically and turned at right angles. Travel in a horizontal direction must be accompanied by travel in a vertical direction, and the total distance traveled is the sum of the horizontal and vertical segments. He further noted that past research on police patrolling in urban areas had shown that the average distance traveled by a patrol car responding to a call in either direction was one third of the dimensions of the sector, or x/3 and y/3. He also explained that the travel time it took to respond to a call (assuming that a car left immediately upon receiving the call) is simply the average distance traveled divided by the average travel speed.

Ms. Maris told Sergeant Davies that now that she understood how average travel time to a call was determined, she could see that it was closely related to the size of the patrol area. She asked Sergeant Davies if there were any restrictions on the size of the area sectors that cars patrolled. He responded that for their city, the department believed that the perimeter of a patrol sector should not be less than 5 miles or exceed 12 miles. He noted several policy issues and staffing constraints that required these specifications. Ms. Maris wanted to know if any additional restrictions existed, and Sergeant Davies indicated that the distance in the vertical direction must be at least 50% more than the horizontal distance for the sector. He explained that laying out sectors in that manner meant that the patrol areas would have a greater tendency to overlap different residential, income, and retail areas than if they ran the other way. He said that these areas were layered from north to south in the city. So if a sector were laid out east to west, all of it would tend to be in one demographic layer.

Ms. Maris indicated that she had almost enough information to develop a model, except that she also needed to know the average travel speed the patrol cars could travel. Sergeant Davies told her that cars moving vertically traveled an average of 15 miles per hour, whereas cars traveled horizontally an average of 20 miles per hour. He said that the difference was due to different traffic flows.

*Develop a linear programming model for this problem and solve it by using the graphical method.*