Problem 2: Assume that a computer can perform 10^6 multiplications per second. Estimate the time that it would take to evaluate the determinant of a 100×100 matrix M. You should not assume that you know the LU factorization of M but that you obtain it in approximately $(1/3)n^3$ operations (See Table 6.1 in page 277 of the text.)

Table 6.1. Operations Count for $A \to U$

Step	Additions	Multiplications	Divisions
1	$(n-1)^2$	$(n-1)^2$	$n-1$
2	$(n-2)^2$	$(n-2)^2$	$n-2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n-1$	1	1	1
Total	$\frac{n(n-1)(2n-1)}{6}$	$\frac{n(n-1)(2n-1)}{6}$	$\frac{n(n-1)}{2}$

Problem 3: Let $a = 10^{-10}$ and consider the system

$$\begin{pmatrix} 1 & 1 \\ a & 0 \\ 0 & a \end{pmatrix} X = \begin{pmatrix} -a \\ 1+a \\ 1-a \end{pmatrix}$$

Part I: Set up the normal equations $A^T AX = A^T B$. Verify by hand that $X = [1,-1]^T$ is the solution of the normal equations, corresponding to the least square solution.
Part II: Try computing the least square solution using MATLAB with the normal equations. Compare the solution with the exact solution obtained in Part I.
Part III: Use MATLAB `qr` command to compute the least square solution by the QR method and compare again with the solution obtained in Part I.

Problem 4: Consider the non-linear system

$$x^2 + y^2 = 4, \quad x^2 - y^2 = 1,$$

Part I: Check that the exact solutions are the four points with coordinates $x = \pm\sqrt{2.5},\ y = \pm\sqrt{1.5}$.
Part II: Using as initial guess $x_0 = 1.6,\ y_0 = 1.2$ perform one iteration using Newton's method.
Part III: Write a MATLAB program to implement Newton's method.
