Assignment

This assignment is to be completed and submitted with full solutions when reporting to write the Module Test.

1. Apply the rules of exponents to simplify the following:

   a) $-(-2x^3)^5$

   b) $-(20t)^{-2}$

   c) $16x^4(2x^3y^{-4})^{-5}$

   d) $\dfrac{(P+Q)^{-5}}{(P+Q)^3(P-Q)^{-1}}$

2. Write each of the following in a radical form:

   a) $(3x)^{1/5}$

   b) $8x^{3/4}$

   c) $x^{1/4}y^{-1/5}$

   d) $\left((x^2+1)^{1/2}\right)^{1/3}$

   e) $(P^{1/3}+Q)^{-1/2}$

3. Express each of the following in exponent form.

   a) $\sqrt[4]{\dfrac{12y^4}{z^6}}$

   b) $\sqrt[3]{\dfrac{27(x-8)^3}{(y+2)^6}}$

   c) $\sqrt{\sqrt[4]{5x^3}}$

   d) $\dfrac{(3t+s)^2}{\sqrt{3t+s}}$

4. Simplify.

   a) $\sqrt{150x}-\sqrt{54x}+\sqrt{294x}$

   b) $A\sqrt{2}+B\sqrt{8}-C\sqrt{50}$

5. For each of the following fractional terms, multiply the numerator and denominator by the conjugate of the denominator, and simplify.

   a) $\dfrac{7+y}{\sqrt{2y}-2}$

   b) $\dfrac{8R}{1-\sqrt{R}}$

6. Simplify with no negative exponents.

   a) $(15x^4y^3)(30y^4z)$

   b) $-5PQ^7(3Q^2P^{-2})^3$

   c) $y^2x^2(x^2 - 3x + 1)$

   d) $(15x^4y^3 + 30y^4z)(2x^{-3}y)$

   e) $(3t - 1)(4t^3 - 5t + 11)$

   f) $x^4 + 2x^2 - 6(-3x^2 + 2)$

   g) $(2a - 3b)(4a^2b + 3b^3)$

   h) $\dfrac{-21xy^3}{(7x^3y)^2}$

   i) $\dfrac{4x^5 - 8x^2y^2 + 3xy^3}{24xy^2}$

7. Simplify.

   a) $(3x^2 + 4x - 2) \div (x + 2)$

   b) $(8 - 2x^2 + x) \div (x + 1)$

   c) $\dfrac{3R^2 - 4R - 4}{2 - R}$

8. Factor the following where possible.

   a) $x^5 - 6x^4 + 2x^3$

   b) $7x^3(y + 1)^2 - 2x^4(y + 1)^2$

   c) $3x^2 - 7x + 4$

   d) $8x^2 + 31xy - 4y^2$

   e) $5x^2 - 2x - 1$

   f) $4x^2 + 48xy - 112y^2$

   g) $\dfrac{1}{64}x^2 - 9t^2$

9. Rewrite and simplify where possible by factoring (-1).

   a) $\dfrac{3x - 4}{8 - 6x}$

   b) $-\dfrac{x^2 - 1}{(1 + x)^2}$

   c) $\dfrac{x^2 - 3xy - 4y^2}{4y - x}$

10. Supply the missing terms so that the fractions are equivalent.

a) $\dfrac{4x + 1}{2x^2} = \dfrac{?}{6x^3 - 8x^2}$

b) $\dfrac{2}{y - x} = \dfrac{?}{x^2 - y^2}$

c) $\dfrac{3}{xy - y^2} = \dfrac{?}{y(x^2 - y^2)}$

11. Reduce to lowest terms.

a) $\dfrac{y^3 - 5y^4}{2y^2}$

b) $\dfrac{2x - 2y}{8xy}$

c) $\dfrac{15x^2y^3 + 5xy}{20xy^2}$

d) $\dfrac{5m^2 - 5}{3m^2 - 7m + 4}$

e) $\dfrac{6x^2 - 4x - 2}{27x^3 + 18x^2 + 3x}$

12. Simplify.

a) $\dfrac{3x^3(4x + 3y^2)^2 + x^5(4x + 3y^2)}{4x + 3y^2}$

b) $\dfrac{5(3x - 1)^{3/4} + 25(3x - 1)^{-1/4}}{10(3x - 1)^{3/2}}$

13. Simplify.

a) $\left(\dfrac{25x^2 - y^2}{16x^2z - 9z}\right)\left(\dfrac{4xz - 3z}{5x + y}\right)$

b) $\left(\dfrac{27P^2R^2 - 3R^2}{3P^2 - 2P - 1}\right) \div \left(\dfrac{15PR - 5R}{3P + 1}\right)$

c) $\left(\dfrac{64r^2 - 16}{2r^2 - 3r + 1}\right)\left(\dfrac{r - 1}{2r^2 + 3r + 1}\right)$

14. Combine and simplify.

a) $\dfrac{1}{2x} + \dfrac{5}{3x}$

b) $\dfrac{6}{x + 1} + \dfrac{18}{-x - 1} + \dfrac{2}{1 + x}$

c) $\dfrac{3}{x^2} + \dfrac{x - 2}{xy} - \dfrac{5}{y^2}$

d) $\dfrac{2x}{x^2 + 4x + 3} + \dfrac{3}{x + 1} - \dfrac{5x}{x + 3}$

15. Simplify the following complex fractions.

a) $\dfrac{\dfrac{R^2 - r^2}{(R + r)^2}}{\dfrac{R - r}{6R + 6r}}$

b) $\dfrac{\dfrac{3}{2x} - \dfrac{5x}{y}}{\dfrac{x}{x^2 + 3}}$

c) $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{x + y}{xy}}$

d) $\dfrac{\dfrac{3k}{k - 2}}{\dfrac{6mk^2}{k^2 - 4}}$

16. The following expression arises in differential calculus:

$$\dfrac{\dfrac{1}{x + h} - \dfrac{1}{x}}{h}$$

a) Simplify this complex fraction.

b) Determine the value of the simplified expression when h = 0.