

B. Probability

N.B. $\mathbb{E}[f(x)]$ is the expectation of the function $f(x)$, given some probability density function $p(x)$.

1. (a) Consider the probability density function $p(x; \lambda)$

$$p(x; \lambda) = \begin{cases} A\lambda x \exp(-\lambda x^2) & x \geq 0 \\ 0 & x < 0 \end{cases}$$

where $\lambda (> 0)$ and A are both constants. Calculate the value of A .

- (b) Show that the even moments of $p(x; \lambda)$ are given by ,

$$E[x^{2n}] = \frac{n!}{\lambda^n}, \quad n = 0, 1, 2, \dots$$

This can be done by recursion.