UNIVERSITY OF NATAL
School of Electrical and Electronic Engineering
CONTROL SYSTEMS - DNE3CO2

0) INTRODUCTION

This is a second contact course in Control Systems. The broad aim of the course is to further develop skills in the domain of control engineering.

1) An engineering design perspective of control systems and feedback systems in particular:
Block diagrams, feedback and feedforward systems - basic understanding of system configurations.
Algebraic notions on the purpose of feedback - simple examples of the benefits and costs of feedback.
System specifications in the time and frequency domain - use of low order models for specifying desired performance.

2) Linear system stability: Because of the lack of precision in everyday language, we have only a vague idea of what stability means. (Think of an unstable political situation - what does this mean?) For engineering application we will require an exact, testable definition of stability. We will obtain this for linear systems in this course. We will find that for design of practical dynamic systems, stability alone will not be an adequate measure of performance and relative stability concepts will be developed. This chapter includes the important tools of Nyquist stability analysis and the root locus.

3) System compensation: In this section of the course we will develop some understanding of feedback systems and skills in the design of feedback controllers. We will investigate disturbance rejection problems and tracking problems. We will discuss common industrial controller configurations such as the PID controller.

1. Engineering systems and feedback ideas (Ogata 1-1, 1-2. F&P Ch. 1)

1.0 Introduction

Examples of technical systems in which feedback control is used:

i) Industrial
- boiler pressure and level control
- superheater temperature control
- temperature control in furnaces, ovens etc.
- composition in chemical processes, viscosity, pH density pulp and paper, oil refineries
- level control in tanks
- motor speed control
- robot control (welding, painting, packing,…)

ii) Commercial - home appliances - irons to CD players (speed + head location), auto speed and ignition control.

iii) Military
- killing people and destroying property.
- (fire control systems, missiles etc.)

Specifications

In design of nearly all technical systems the specifications represent a trade-off between performance and cost - VW Beetle versus Porsche. We would often like to produce the "best" performance that is possible for the given hardware. Of course, the control engineer may impact on the design to ensure that the desired performance can be obtained under dynamic conditions.

Control objectives

1) *Tracking*: Follow an external reference signal, r(t), (within some bands without saturating the input u(t)). Examples:

   (a) temperature profile in annealing, (b) welding robot (c) aircraft automatic landing system.

2) *Disturbance rejection (regulation)*: Reduce/eliminate the effect of external disturbances, d(t) on the system output. Examples: (a) Car speed control with wind and road gradient changes (b) "Evening tank" in chemical processes (c) Wow and flutter removal in CD player (d) Air conditioning plants. Disturbances enter at the input or at some point within the plant.

3) *Reducing effects of process uncertainty*: Most processes have parameters that change with time and operating conditions. Additionally we apply linear techniques to non-linear processes. Feedback can be used to reduce the effect of process uncertainty on the system response. *Examples* (a) Op Amps (open loop gain of μ741 is between 10^6 and 10^2) (b) Chemical processes (c) Aircraft - Mach number and air pressure/altitude (d) Anaesthetic and drug administration.

4) *Stabilisation*: Not too many processes are designed to be unstable. Thos that are must be operated with feedback. Examples (a) Exothermic chemical reactions (e.g. $N_2 + 3H_2 \rightarrow 2NH_3$ + dramatic amount of energy) (b) Bio-reactions (c) Rockets (d) Military aircraft (e) Nuclear reactors.

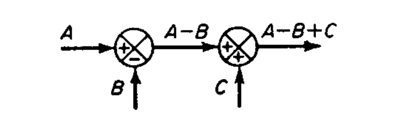
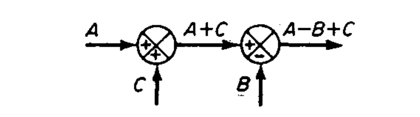
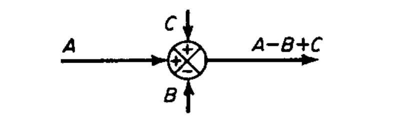
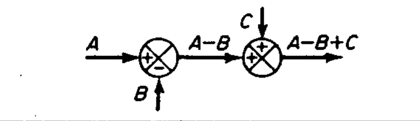
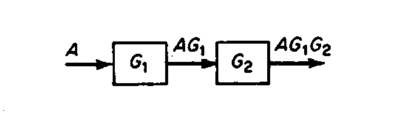
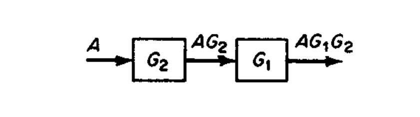
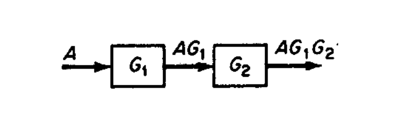
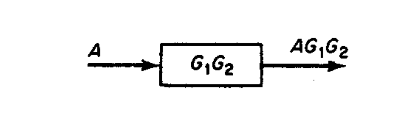
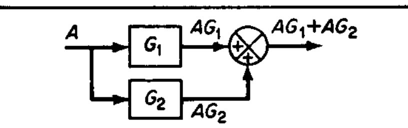
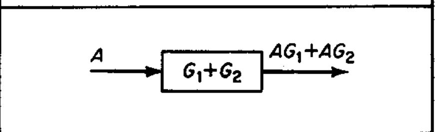
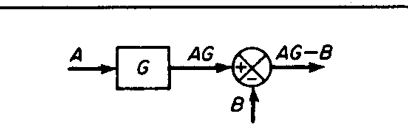
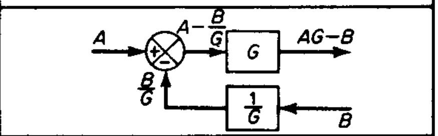
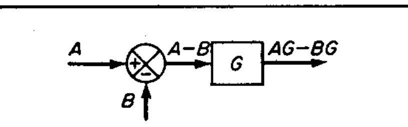
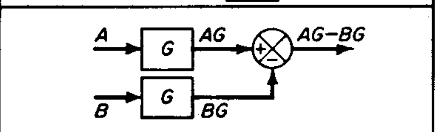
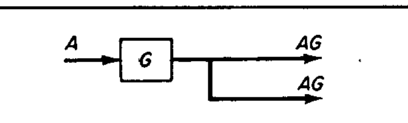
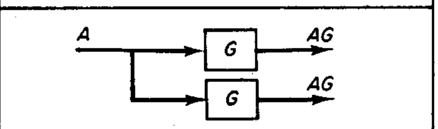
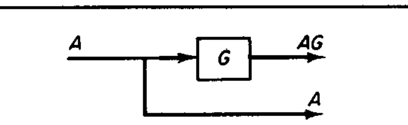
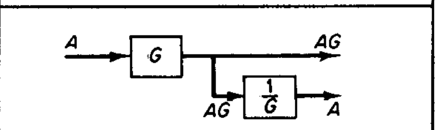
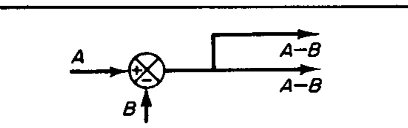
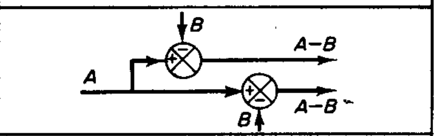
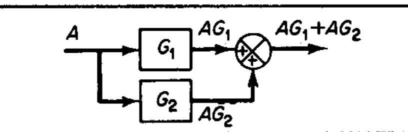
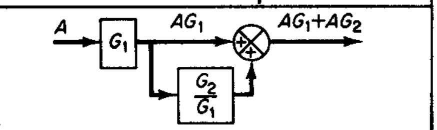
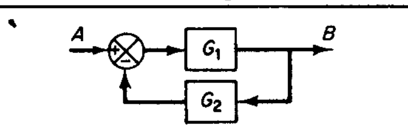
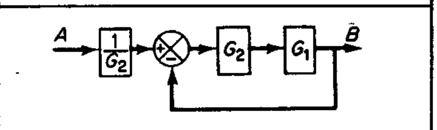
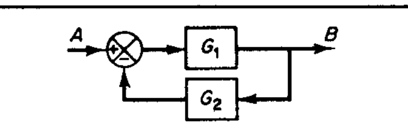
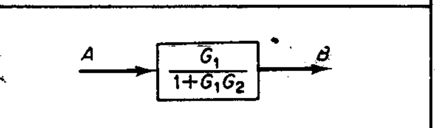
Tuts - Ogata A-1-1 to A-1-3, B1-1 to B-1-4.

1.1) Block diagram reduction

Because we use the Laplace (frequency) domain, output of a block is simply the product of the input and the block's transfer function. Block design reduction follows (with practice) from this. You may also use signal flow analysis (Mason's rule) for more complicated systems.

Tuts, Ogata A-3-1, A-3-2, A-3-4, A-3-7, A-3-9, A3-11, B-3-1, B-3-2

Table 1–3 Rules of Block Diagram Algebra

	Original Block Diagrams	Equivalent Block Diagrams
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

ref. Ogata.

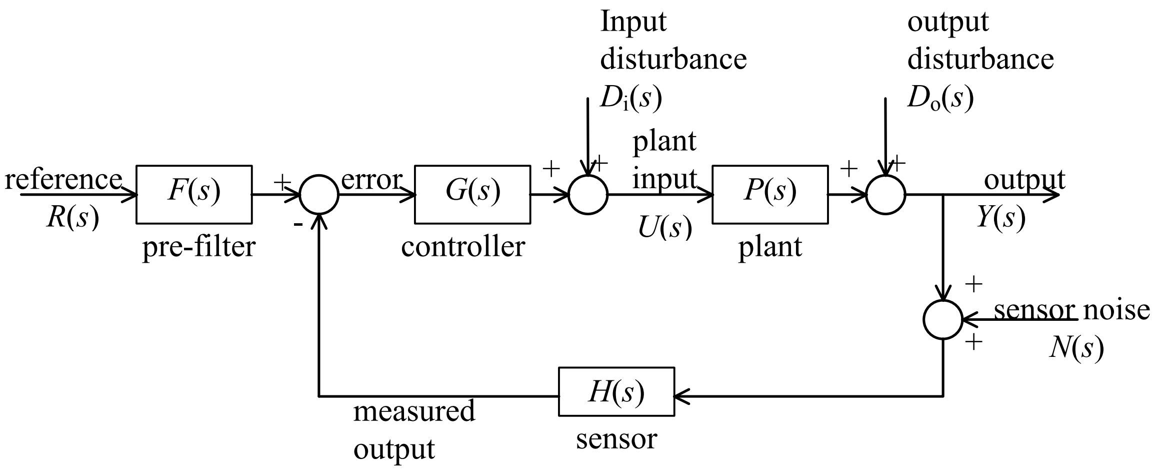


Figure 1 - Feedback system components

For input disturbance, $P^* = P$. For output disturbance, $P^* = 1$. The disturbance may enter through part of the plant. For the system in Figure 1, we have,

$$(1 + GPH)\, Y = FGP\, R + P^*\, D - GPH\, N$$

$$(1 + GPH)\, U = FG\, R - GH\, (N + P^*\, D)$$

This gives transfer functions (defined for other external signals = 0) of

$T_{Y/R} = \dfrac{FGP}{1+GPH} = \dfrac{F}{H}\dfrac{L}{1+L}$ (reference $\rightarrow$ output)	$T_{Y/D} = \dfrac{P^*}{1+GPH} = \dfrac{P^*}{1+L}$ (disturbance $\rightarrow$ output)
$T_{U/R} = \dfrac{FG}{1+GPH} = \dfrac{FG}{1+L}$ (reference $\rightarrow$ input)	$T_{U/N} = \dfrac{-GH}{1+GPH} = \dfrac{-GH}{1+L}$ (sensor noise $\rightarrow$ input)

Observe the effect that high gain has on each of these transfer functions. For L>>1,

$T_{Y/R} \approx \dfrac{F}{H}$ "Ideal" tracking by design of F/H	$T_{Y/D_o} \approx 0$ "Ideal" disturbance rejection
$T_{U/R} \approx \dfrac{F}{PH}$	$T_{U/D_o}, T_{U/N} \approx \dfrac{-1}{P}$

1.2) Algebraic notions on the purpose of feedback

Consider for the moment that all blocks are simple gains.

a) *Disturbance rejection*

$p = 2$, $p^* = 1$, $h = 1$

What gain, g is required to keep effect of $d = 3$ on output ≤ 0.1?

$$T_{Y/D} = \left|\frac{p^*}{1+pgh}\right| \leq \frac{0.1}{3}$$

$$\xrightarrow{g \geq 14.5}$$

b) *Uncertainty reduction for tracking*

$1 \leq p \leq 10$, $h = 1$.

i) What g is required to get tracking error of 5% if $f = 1$

$$0.95 \leq \left|\frac{gpf}{1+gph}\right| \leq 1.05$$

$$gp \geq \frac{0.95}{0.05} \Rightarrow g \geq 19 \text{ for worst case } p = 1$$

ii) As above, but design f as well:

$$0.95 \leq \left|\frac{fgp}{1+gph}\right| \leq 1.05$$

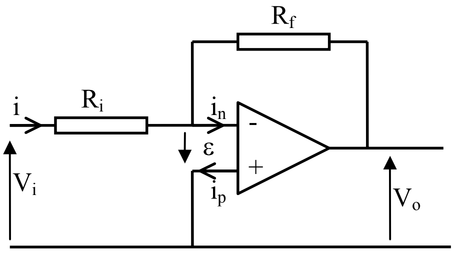
$0.95 \leq (f - 0.95)\, gp$, and $(f - 1.05)\, gp \leq 1.05$

The lowest gain design is found with

$0.95 = (f - 0.95)\, g\, p_{min}$ and $(f - 1.05)\, g\, p_{max} = 1.05$

$f = 1.063$, $g = 8.41$ (ie 44% of previous value).

Performance of an op-amp



$V_o = A\,\varepsilon$

$$\frac{V_i+\varepsilon}{R_i} = -\frac{V_o+\varepsilon}{R_f}$$

$$\frac{V_1}{R_i} = -\frac{V_o}{R_f} - \frac{V_o}{AR_f} - \frac{V_o}{AR_i}$$

$$= -V_o\left(\frac{1}{R_f}\left(1+\frac{1}{A}\right)+\frac{1}{R_iA}\right)$$

$$\frac{V_o}{V_i} = -\frac{R_f}{R_i}\bigg/\left(1+\frac{1}{A}\left(1+\frac{R_f}{R_i}\right)\right)$$

$$= -\frac{R_f}{R_i}\frac{A\left(\frac{R_i}{R_i+R_f}\right)}{1+A\left(\frac{R_i}{R_i+R_f}\right)} = -\frac{R_f}{R_i}\frac{pg}{1+pg}$$

$p = A$, $g = R_i / (R_i+R_f)$. If $R_i = 1$ kΩ, $R_f = 1$ MΩ, $A \in [10^5 ... 10^6]$, $g = 9.99\times10^{-4}$

$$\frac{V_o}{V_i} = -1000\times[0.9901 \ldots 0.999]$$

(Worst case ~ 1% gain error and 1% variation with gain). The solution is to use a better quality op amp or achieve the required gain in stages.

1.3) Specifications in the time and frequency domain

Engineering expectations are usually expressed in the time domain but our design approach will be in the frequency domain. We will use (for example) step response of low order systems to translate specifications to the frequency domain. In practical systems, other signals may be more appropriate.

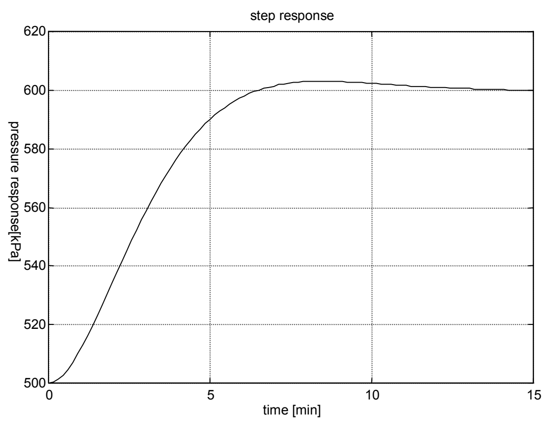
Examples

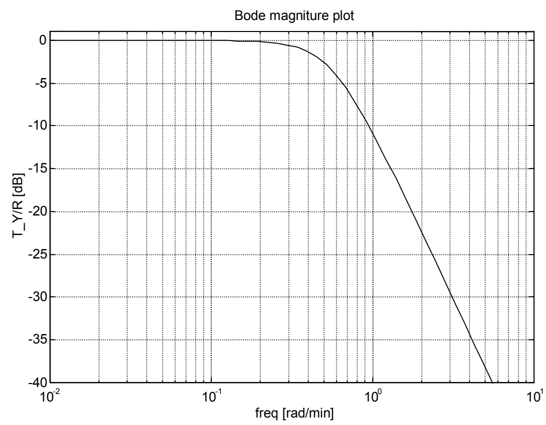
1) Boiler pressure: The pressure in a boiler must follow a step change in pressure reference from 500kPa to 600kPa with no more than 3% (3kPa) overshoot and achieve 90% of the final value in 300s (5 minutes). Find the frequency domain specification for $T_{Y/R}$.

Solution: Choose 2nd order behaviour. $M_p = e^{-\pi\zeta / \sqrt{1-\xi^2}} \Rightarrow \zeta = 0.745$.

Simulation with ω_n = 1 rad/s (»step(1,[1,2*0.745,1])) gives $t_{90\%}$ = 2.78s. This can be found to about 5% accuracy by inspection of normalised second order step responses.

For $t_{90\%}$ = 5, ω_n = 2.78/5 = 0.556 rad/minute





Desired response

»step(100,[1/0.556^2,2*0.745/0.556,1])

Required transfer function

$$T_{Y/R} = \frac{1}{(s/0.556)^2 + 2\times 0.745(s/0.556)+1}$$

»bode(1,[1/0.556^2, 2*0.745/0.556, 1])

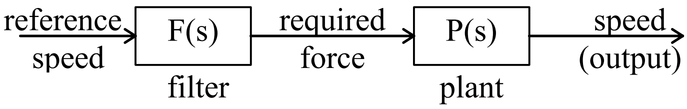
In a great number of reference tracking problems, low pass behaviour is expected - We do not expect any response initially because the physical system has conserved variables (energy in the water/steam in the above example) that would need infinite (impulsive) inputs to change instantly. We do require that the system output eventually is at or near the reference input for tracking. Most control text books labour low pass responses and ignore other possibilities. Typically, we could specify the following parameters for second order step responses: Delay time, rise time, peak time, settling time, % overshoot, steady state error (position). The steady state error may also be specified for ramp and parabolic reference signals - the velocity and acceleration errors.

Tuts Ogata A-4-3 (Useful for CO2 laboratory), A-4-4, A-4-6 to A-4-9

Example 2

Consider the problem of speed control in a motor car described by $m\dot{v} + \mu v = f$, $T_{V/F} = \dfrac{1}{ms + \mu}$.

If things were this simple we could achieved any desired behaviour by filtering. For example if the response to a reference, R(s) was required to be $T_{V/R} = \dfrac{1}{s\tau + 1}$, the following system could be used.



We would choose $F(s) = \dfrac{ms + \mu}{s\tau + 1}$. This is approach impractical for the following reasons.

1) The mass is unknown and changes as passengers and luggage are loaded. The mass of fuel changes while the car is being driven.

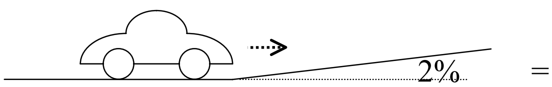
2) The friction is non-linear, being is made up of rolling friction (which depends on the road surface and tyre condition) and drag (which depends approximately on the square of the speed).

3) There are unknown wind and road slope forces acting on the vehicle. In normal speed control problems, the reference stays constant and the control is required only regulate out the effect of these disturbances

4) The relationship between throttle position and engine torque is not known and has its own dynamics.

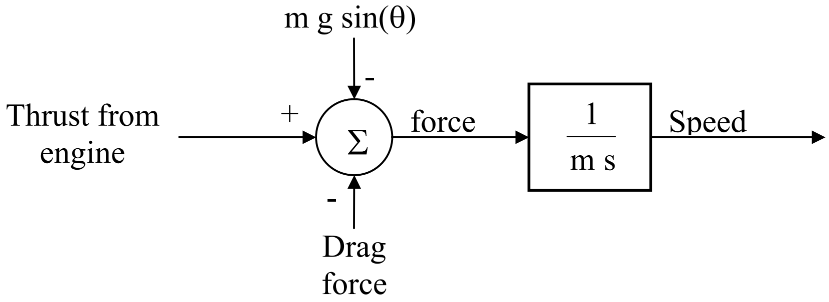
5) As $s \to \infty$, $F(s) \to m/\tau$ which suggests that for a step change in speed we will require an immediate change in the torque produced by the engine. Due to inertia of the engine, this is not reasonable.

Specific numerical example: Speed control when encountering a 2% change in road gradient.



User requirements: i) not more than 2 km/h speed loss
ii) no steady state error

Force balance is:



Here the disturbance acts before the final output and without feedback, we anticipate a deceleration of

$g \sin(\theta) = 0.196 \text{m/s}^2 = 0.705 \text{ km/h/s}$

With H=1, the transfer of the step change in force to the output for this problem is

$$T_{Y/D} = \frac{P}{1+GP} = \frac{P}{1+L}$$

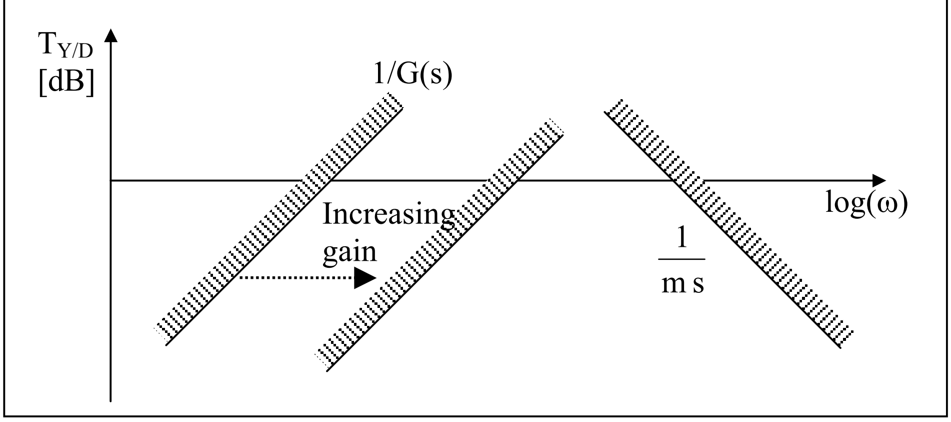
For zero steady state error, G must have an integrator (k/s) so that $G(0) \rightarrow \infty$

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s\,Y(s)$$

$$= \frac{P(0)}{1 + P(0)G(0)}$$

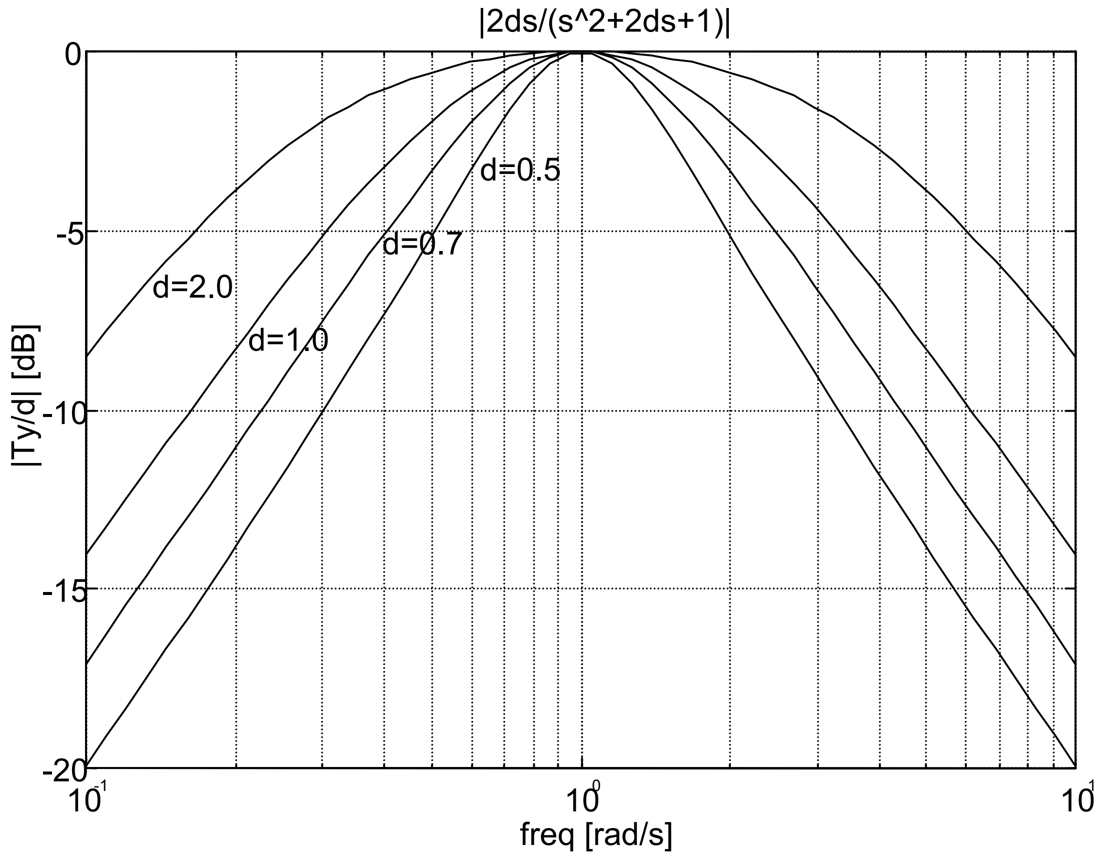
- $T_{Y/D}$ is constrained at low frequency where $|GP| >> 1$ by $T_{Y/D} \approx 1/G(s) \approx s/k$. This can be designed.

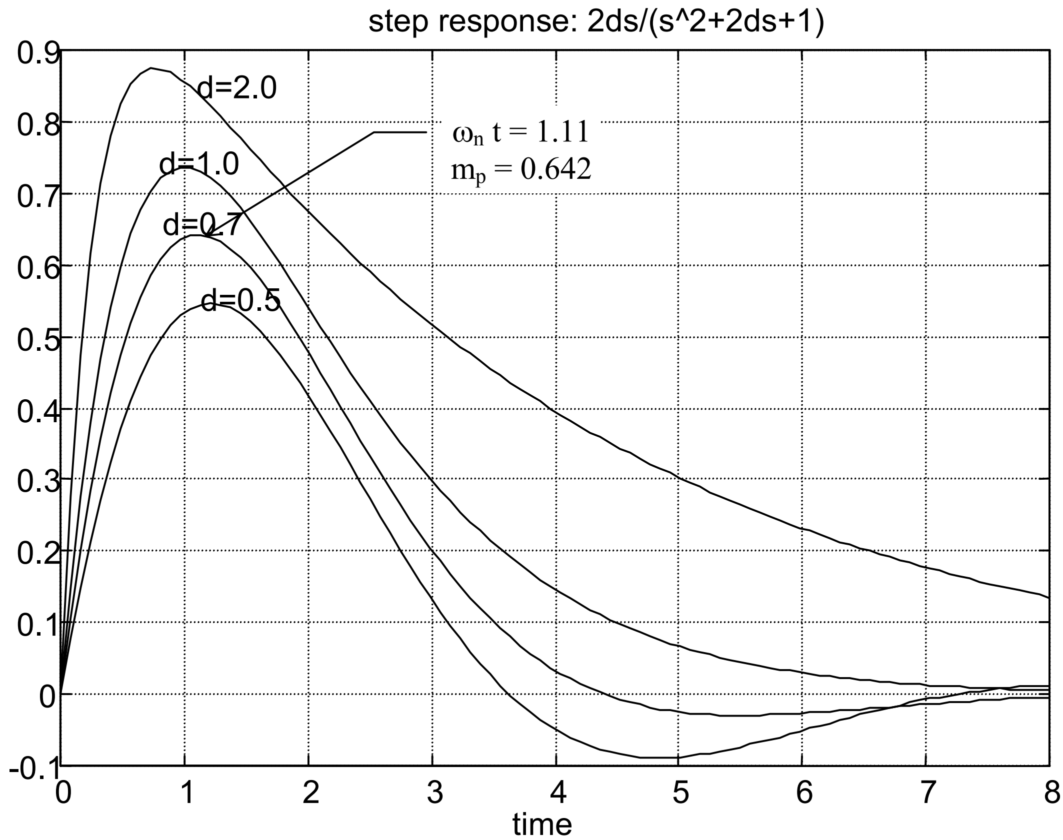
- $T_{Y/D}$ is constrained at high frequency where $|GP|<<1$ by $T_{Y/D} \approx P \approx \frac{1}{m\,s}$. This is fixed by the plant design.
- By design around the meeting of the asymptotes, we can also modify the performance.
- For robust stability (to be discussed later) we usually constrain $\left|\frac{1}{1+L}\right| \le \gamma$ dB, never more than 6dB (2) but more usually 2-3dB (1.25-1.4)



Recall the initial value theorem $\left(\lim_{t\to 0} y(t) = \lim_{s\to\infty} Y(s)\right)$ and final value theorem $\left(\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)\right)$. The above is a band pass response - initially (i.e. at high frequency) low gain because of the plant dynamics and finally low gain because of the controller action. We can compare these constraints to second order band pass responses shown below:

$$T_{Y/D} = k\frac{2\xi s/\omega_n}{(s/\omega_n)^2 + 2\xi s/\omega_n + 1}$$ (with mid frequency gain of k)





For the car problem, if m=1000kg, ζ=0.7 (chosen by the designer from the normalised curves above to give recovery with more or less the same speed as the manifestation of the error and not too much oscillation, subject to client approval!), the response is

$$T_{Y/D} = k\frac{2\times 0.7\times s/\omega_n}{(s/\omega_n)^2 + 2\times 0.7\times s/\omega_n + 1},$$

with $k \times 2 \times \zeta \times \omega_n = 1/m$ to have the correct high frequency gain.

The time response shown will be simultaneously scaled in amplitude by k while the time will be scaled by $1/\omega_n$ (i.e. if we want shorter time to peak (faster response), we make ω_n larger.

E.g. for a step of 2% (θ=atan(0.02), step in force is $m \times g \times \sin(\theta)$= 196 N):
a) For a 2 km/h = (2/3.6) m/s worst drop in speed we must choose

$$k = \frac{(2/3.6)}{0.642\times m\times 9.8\times \sin(\theta)} = 4.416\times 10^{-3} \quad \text{and } \omega_n = 0.162 \text{ rad/s}$$

b) For t_p = 30s, ω_n = 1.11/30 = 0.037 rad/s. This gives k = 19.3×10^{-3} and worst speed error of
$k \times 0.642 \times m \times 9.8 \times \sin(\theta) = 2.43\text{m/s} = 8.74\text{km/h}$
Responses are shown below.

```
» wn=0.037;
» k=19.3E-3;
» u_max=9.8*sin(atan(0.02))*1000;
» step(u_max *[k, 0], [1/wn^2, 2*d/wn, 1]);
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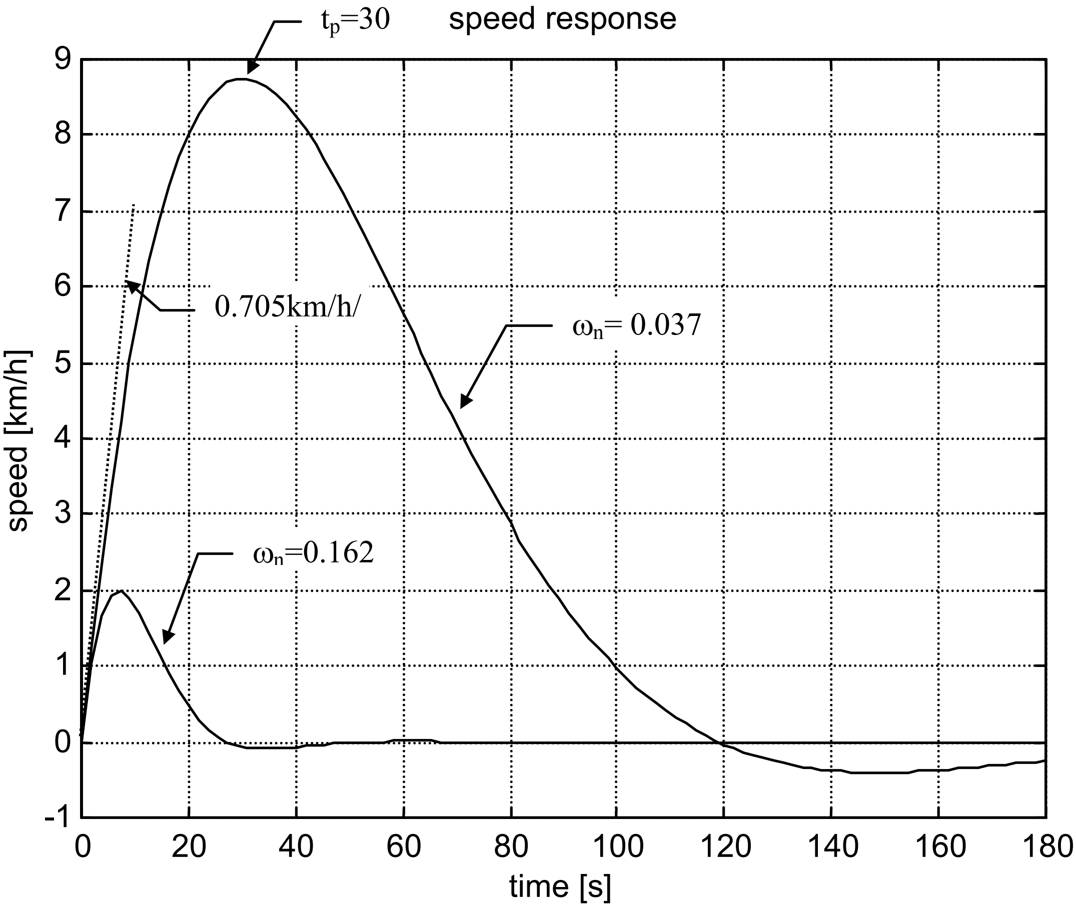
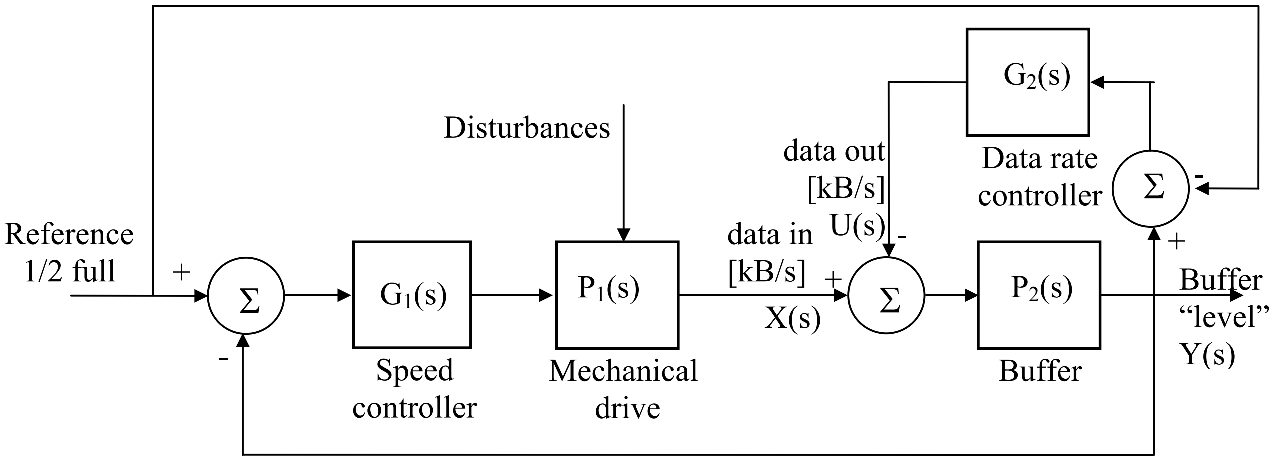
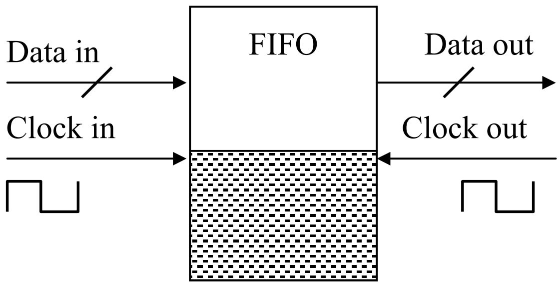


t_p=30
speed response
0.705km/h/
ωn= 0.037
ωn=0.162
speed [km/h]
time [s]
9
8
7
6
5
4
3
2
1
0
-1
0
20
40
60
80
100
120
140
160
180

Example 3 Buffer in digital system. A digital tape drive has the block diagram shown below.



As the mechanical system is slow, digital data is buffered in a First-In-First-Out (FIFO) buffer. The design requirements will be that the buffer must absorb sudden changes in data rate and give the "smoothest possible" output data rate by the *slowest possible* control action. NB The electronics people are interested in the output data, not the buffer level.



For example, a 64k buffer is used and input data rate is between 3k/s and 6k/s. Design to not overflow buffer on 4.5k to 6k transition.

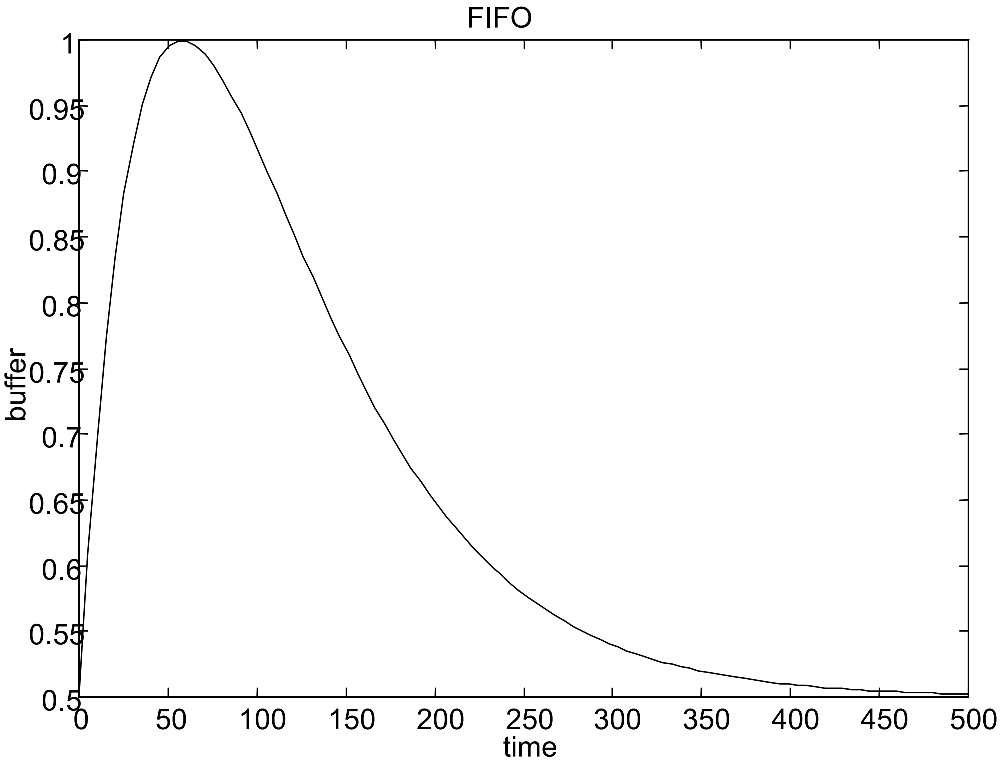
Solution: Choose ζ=1.0 for overdamped response. From normalised plot, peak is 0.736 at t=1.
$P_2(s) = 1/(64s)$ if input is in k/s and buffer level is between 0 (0k) and 1 (64k).

High frequency gain of $T_{Y/X} = \dfrac{P_2}{1+G_2P_2} = k\dfrac{2\zeta s/\omega_n}{(s/\omega_n)^2 + 2\zeta s/\omega_n + 1}$ is $\dfrac{k\,2\zeta\omega_n}{s} = \dfrac{1}{64s}$

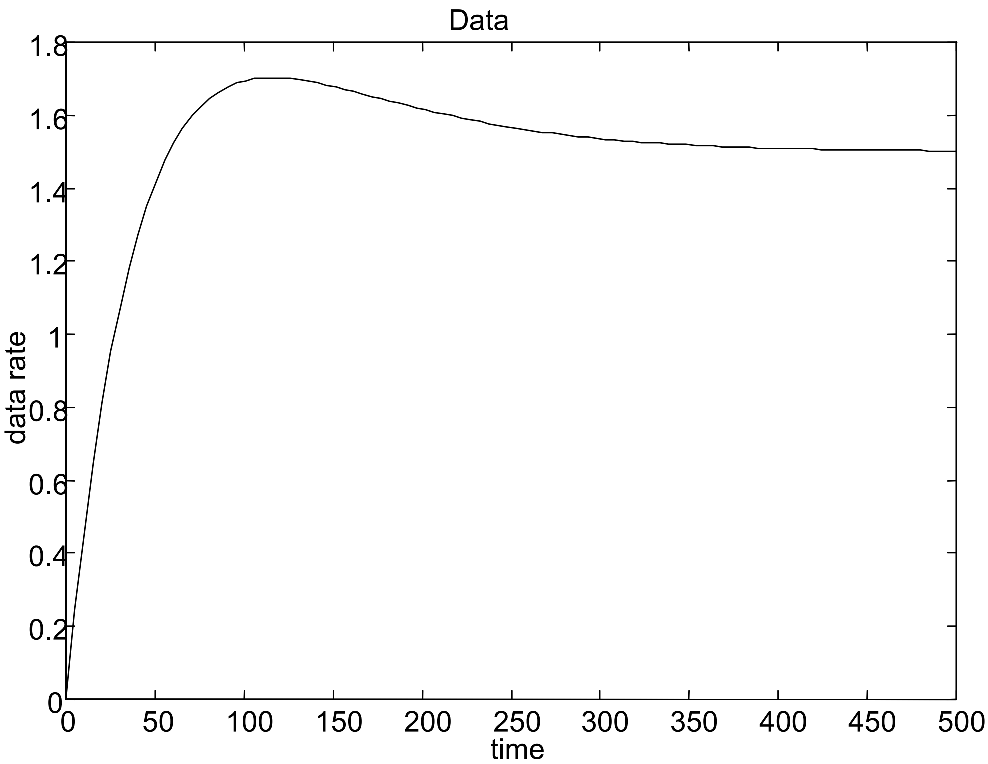
We require $y_{max} = 0.5$ for input of 1.5, i.e. gain of 1/3 = k × 0.736 => k = 0.4529.

From high frequency, $\omega_n = 0.01725$ rad/s

Data rate out is $T_{U/X} = \dfrac{G_2P_2}{1+G_2P_2} = 1 - \dfrac{T_{Y/X}}{P}$ - this avoids explicitly solving for $G_2(s)$.




```
» k=0.4529;   wn=0.01725;
» y=0.5+step(1.5*[2*k/wn 0],[1/wn^2, 2/wn, 1],t);
```




```
» y=1.5*(1-step([64*2*k/wn 0 0],[1/wn^2, 2/wn, 1],t))
```

Formulas for tp and yp for band-pass systems with a unit step input:

$$T_{Y/D} = k\frac{2\xi s/\omega_n}{(s/\omega_n)^2 + 2\xi s/\omega_n + 1}$$

For d(t)=σ(t), a unit step, D(s) = 1/s

$$y(t) = \frac{k2\zeta}{\sqrt{1-\zeta^2}} e^{(-\zeta\omega_n t)} \sin(\omega_d t)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

For turning points,

$$\frac{dy}{dt} = \frac{k2\zeta}{\sqrt{1-\zeta^2}} e^{(-\zeta\omega_n t)} \left(-\zeta\omega_n \sin(\omega_d t) + \omega_d \cos(\omega_d t)\right) = 0$$

$$\tan(\omega_d t_p) = \frac{\sqrt{1-\zeta^2}}{\zeta} \quad \text{or} \quad \sin(\omega_d t_p) = \sqrt{1-\zeta^2} \quad \text{or} \quad \cos(\omega_d t_p) = \zeta$$

The peak value is

$$y(t_p) = k2\zeta e^{\left(-\operatorname{acos}(\zeta)\zeta/\sqrt{1-\zeta^2}\right)}$$

which depends only on ζ and k.

$2\zeta e^{\left(-\operatorname{acos}(\zeta)\zeta/\sqrt{1-\zeta^2}\right)}$ is the number we look up on the normalised charts.